33. SAMPLING CONSIDERATIONS FOR REGIONAL GEOCHEMICAL SURVEYS

Project 750039

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Abstract

A number of aspects of sample design are reviewed in the context of regional geochemical studies. Particular attention is paid to optimal sample spacing and collection so that surveys can be executed at minimum cost. Procedures for undertaking quality control monitoring of sampling and analytical variability as integral parts of geochemical surveys are discussed in the light of new methods for carrying out analysis of variance. Finally, an approach is proposed for the estimation of survey efficiency. This proposal draws on aspects of both previously discussed topics and should aid the objective appraisal of survey results in mineral resource studies.

Introduction

This report documents some of the more interesting topics encountered by the author in studying and designing sampling programs for regional and exploration geochemistry. None of the projects from which the following observations were made were specifically sampling studies, however, they have all included sampling design stages as an integral part.

Before commencing a sampling program it is imperative to know clearly what the objective program will be in terms of the ultimate data requirements, these may be needed to assist some decision making process or map the variability in the area of interest. In this context, it has been found useful to divide sampling programs into two broad categories. Firstly, 'exploration' mode, where the objective is to cover a geographic or geological unit in the search for targets and then classify the resulting data into 'hits' or 'misses'. The 'hits' will be the focus of further mineral exploration activity, hopefully leading to the discovery of significant mineral occurrences. Secondly, 'evaluation' mode, where the objective is to prepare reliable maps of the variability of some parameter across a geographic or geological unit. Such maps may well be for purposes other than mineral exploration.

One concept which is of importance in sampling design is that of 'target population' and 'sample population'. The objective of a survey is to make statements about a whole area. All the possible samples of the material of interest one could collect from that area form the 'target population', i.e. the 'target population' is the totality about which one finally wishes to make statements. In almost all cases in geology it is physically and economically impossible to collect the 'target population'. What occurs is that according to some rules the geologist has defined a suite of samples is collected which form the 'sample population'. This suite is one of an infinite number that could be collected and is that geologist's realization of an infinite problem. From this 'sample population' will be inferred properties of the 'target population'. This concept of different populations was first introduced to geology by Rosenfeld (1954) and later discussed at length by Krumbein (1960).

The report is divided into three sections on topics related to 'exploration' mode, 'evaluation' mode, and a combined approach which evolves from the previous two sections. In 'exploration' mode the concern in sampling design is to collect sufficient samples to enable decisions to be made with a desired confidence, but not to collect more, as this would be wasteful of resources. Three separate methods will be discussed, the first is purely geometric in concept and assumes a minimum of geological knowledge, the remaining two require more detailed geological knowledge and make genetic assumptions concerning the relationship of mineral occurrences to geological units. In 'evaluation' mode the concern is to so structure the sampling design that it is possible to determine if any regional trends revealed by the survey are real, or could be due to the accumulated effects of sampling and analytical variability. In this sense 'exploration' mode work is focused on pre-survey design, whereas 'evaluation' mode focuses on quality control of the executed survey. The combined approach attempts to mate the geometric 'exploration' mode work with the quality control aspects of 'evaluation' mode and derive some measure of survey effectiveness which will aid decision making in mineral resource studies.

Exploration Mode Considerations

In mineral exploration it is common to have some a priori knowledge of the size of the target one is searching for. In geochemical exploration the target is usually larger than the actual mineral occurrence due to secondary dispersion from the occurrence and the primary halo about it, or in the case of lithogeochemical studies, due to primary dispersion patterns alone. In regional geochemistry, systematic sampling on a square or rectangular grid, as used in soil and some overburden/till surveys, is uncommon. More often some modification is employed, as the 'target population' is not available at all locations due to its very nature, or cover of some sort. For example, in a lake sediment survey all lakes in the area would form the 'target population' and these do not conveniently fall at grid intersections. Similarly, except in the rare cases of total exposure, rock samples are not available at all grid intersections due to lakes or overburden. The common reaction to this problem is to grid the area and collect a single sample from within each grid cell, or close to the grid intersections. In effect the geologist is using a sampling plan similar to one known to statisticians as a stratified random sample design.

Tables have been published by Savinskii (1965) to aid in determining optimal grid sizes, either square or rectangular, given the target size and its orientation relative to the grid. These tables are unfortunately of little use with stratified random sample designs which are commonly employed. An alternative approach which has been employed involved simulation studies (Garrett, 1977), or more recently a direct method, which is described below. The probability that a sample collected on a square grid will fall within the target, P(h), is defined as:

$$P(h) = \frac{At}{Ac}$$

where At is the area of the target, often approximated by a circle or an ellipse, and Ac is the area of the square grid cell.

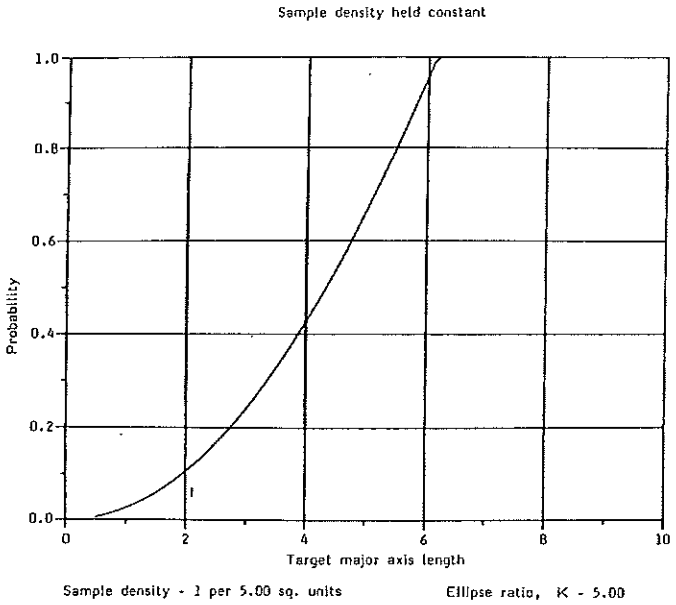

Figure 33.1. Relationship of expectation of a 'hit' to target size.

The probability varies from zero to one, greater values than one, which occur with fine grids and very large targets, being set to one. In the stratified random model the square grid is replaced by a quadrilateral formed by four randomly chosen points in four adjacent square cells. In sampling terms, these four points would represent four sample sites in four adjacent grid cells. An infinite number of quadrilaterals could be so formed. Works by Ghosh (1951), and later Matern (1960), show how the average distance between two randomly chosen points in adjacent squares can be computed. For the case of two adjacent unit size grid cells this average distance, d, is 1.08814 units. Using this distance, the quadrilaterals may be replaced by a single square grid whose sides are increased by the factor d. For instance, if sampling is at 1 sample per 13 km^2 (1 sample per 5 square miles) the grid squares will be $\sqrt{13}$, or 3.61 km, on a side and the effective square size for use in computing the expectation of a 'hit', i.e. that such a design will collect samples from within a target, is 3.92 km on a side (15.39 km^2). This expectation of a 'hit', E(h), can now be computed from a knowledge of the area of the basic sampling unit, Ac, and the factor d. At remains the area of the target.

$$E(h) = \frac{At}{Ac.d^2}$$

With this information in hand the expectation that an individual sample will be drawn from within a target of specified size can be determined. In all likelihood this expectation is an overestimate since it has been assumed that all samples from within targets will be correctly identified. The reality is that due to variability and errors in sampling and analysis there is some probability, less than one, that the 'hit' will be recognized as such. If we define E(h) as the expectation of a 'hit' on purely geometric grounds, and P(r) as the probability of recognition, then the actual expectation of success of the sampling design, E(s), will be

$$E(s) = E(h).P(r)$$

Current research by C.F. Chung of the Geological Survey of Canada is leading towards the definition of the true probability of a 'hit', P(h), for stratified random sample designs, which will in turn allow true probabilities of success to be estimated. In addition, currently there have been no numerical attempts to measure the probability of recognition, P(r), the problem of recognition has been recognized but not quantized objectively. At the present time an interactive graphics computer program, SEARCHSIM, has been developed so that the effects of varying search grid and target sizes can be displayed graphically. Figure 33.1 is a copy of the Cathode Ray Tube graphic output of SEARCHSIM, a sample density of 1 site per 5 square units is assumed and the target approximated by an ellipse five times longer than it is wide. It should be noted that the Y axis is labelled "probability"; this is premature in terms of program development and should more correctly be titled "expectation". The plot shows the relationship between the target major axis length and expectation of a 'hit'. For example, it is seen from the plot, that a target would have to be larger than 6 units long by 1.2 units wide to be hit with better than 0.95 expectation.

The preceding topic assumes very little geological information, only that orientation studies have defined the target sizes and geochemical thresholds in a geographic area. Two alternate approaches may be employed which use different assumptions than those for grid size selection. In the determination of optimal grid sizes it is assumed that the local environment is heterogenous, i.e. targets within an area. In both the next methods to be described it is assumed that the geological or geographic units to be sampled are relatively homogenous, the task being to discriminate the units associated with mineral occurrences from those that are not, i.e. to separate the "productive" from the "barren". The methods focus on aspects and properties of the frequency distributions of the data in two different manners. These approaches assume that there is some genetic link between the mineralization and geological unit in question that will be reflected in the local geochemistry. If such a link is missing the two methods to be described are inappropriate. Comparing these methods with that of optimal grid selection, in the grid case the "hit" is a single sample, or a group of adjoining samples. In the cases to be discussed below the "hit" is a whole suite of samples which represent some specific unit, where the whole unit forms the "hit". It is possible that a subsequent review of the data for the unit may reveal the most favourable areas to commence the next stage of field work, but that is not the prime objective of this type of sampling design.

It has been noted by many workers that geological units characterized by geochemically complex data are often hosts to mineral occurrences, and that where geochemical data lack contrast there are usually no mineral occurrences. This relates to the concept of relief in geochemical data. A numerical expression of relief is the coefficient of variation of the data set, cv; this is usually expressed in percentage terms as

$$cv\% = 100.s/\bar{x}$$

where s is the standard deviation and $\bar{x}$ the mean of the suite of 'sample population' individuals collected from a specific geological unit or area. Coefficient of variation has been used in several studies (e.g. Garrett, 1972; Hornbrook and Garrett, 1976) as a numerical measure of geochemical relief or contrast. General rules of thumb are, that coefficients of variation below 70% correspond to areas of low geochemical relief and low mineral potential; while coefficients above 100% correspond to complex situations, due to heterogeneity of geology or secondary environment, or the presence of mineral occurrences. The coefficient of variation is related to normal distribution theory, very high or very low values indicate that the data are possibly not drawn from a normal distribution (Koch and Link, 1971).

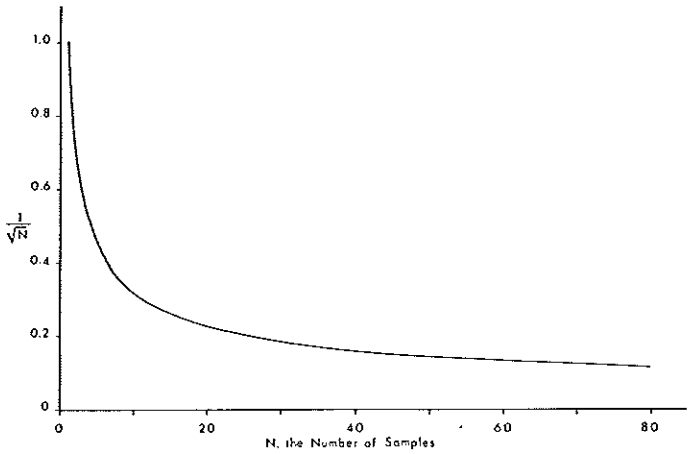

Figure 33.2. Relationship of 1/N to N, the sample size.

The question arises as to how many randomly selected samples should be collected from a geological unit, or area, to obtain precise estimates of the mean and standard deviation to be used to calculate the coefficient of variation. A measure of the reliability of the estimates of the mean and standard deviation are their standard errors. Respectively, $SE_{\bar{x}}$ for the mean, $\bar{x}$, and SE_s for the standard deviation, s, and where N is the number of individual samples collected and anlayzed.

$$SE_{\bar{x}} = \frac{s}{\sqrt{N}} \qquad \text{and} \qquad SE_s = \frac{s}{\sqrt{2N}}$$

From these two equations it is noted that the major control on standard error is the value of N and that the standard error of the standard deviation decreases twice as fast as that for the mean due to the 2N term in the denominator of the former. As the standard error of the standard deviation drops, and improves, twice as fast as the standard error of the mean, it need not concern us furthermore. Certainly if the standard error of the mean is acceptable that for the standard deviation will also be acceptable. Figure 33.2 shows the relationship between the controlling term, $1/\sqrt{N}$, and N. It can be seen that the controlling term drops very rapidly between 1 and 10, begins to level out around 30 and after 60 does not decrease significantly except with very large increases in N. Most statisticians agree that on empirical grounds to exceed 60 is probably passing the point of diminishing returns, i.e. the reductions in standard error do not justify the added costs. Often a figure of 30 is quoted as a more realistic value for N; the author has often used the value of 15 as a bare minimum in terms of the compromise between precise values for the mean and standard deviation and the costs of sample collection and analysis. The choice of the number of samples to be randomly collected to form the 'sample population' will depend upon the particular situation. However, Figure 33.2 puts the impact of the choice into easy graphical perspective, the improvement in precision can only be obtained by increased sampling and analysis, with their accompanying cost.

A technique for selecting optimal sample sizes, which makes no distributional assumptions, is used in the Soviet Union. The method is described by Beus and Grigorian (1977) and is based on tables published by Chernitskii (1957). It is unclear whether these tables are actually in the form reproduced in Beus and Grigorian, or, are simply tables of the cumulative binomial distribution. If the latter is the case, such cumulative tables are available in North America, e.g. from the Harvard Computation Laboratory. In practice a computer program, SMPSIZE, has been written to directly compute the tables necessary for geochemical use, these are presented and discussed below.

The method assumes that prior orientation work in areas, or geological units, that may be classified as 'barren' or 'productive' has been undertaken. The data distributions are plotted as cumulative frequency diagrams or histograms, as illustrated in Figure 33.3. The forms of the distributions are irrelevant, they may be normal, lognormal, mixtures or whatever. A threshold value T is chosen, such that the probability of a sample from the 'barren' population having a value above T is very small. The proportion of the 'productive' population falling above T is then noted. This proportion is the a priori probability of occurrence of a value > T. At this point Table 33.1 is entered at some desired probability of recognition, commonly the 0.95 level (as are the tables in Beus and Grigorian). The last item to be decided upon is how many samples, R, one wishes to have as a minimum to exceed the threshold T. If R, or more, of the N samples that Table 33.1 indicates should be collected exceed T in value then one can classify the unit as 'productive' at the appropriate probability of recognition. As an example, if the a priori probability of occurrence of a value greater than T, denoted, P, is chosen to be 0.10, the probability of recognition is chosen as, 0.95, and at least an R of three individual samples are to exceed T, then a total, N, of 61 randomly selected samples must be collected. If three or more of the 61 samples exceed T the unit is classified as 'productive' and further work planned; otherwise if less than three values exceed T the unit has to be classified 'barren'. In cases where there is a very clear distinction between the 'barren' and 'productive' data sets, such as with some pathfinder elements (e.g. Cd) or elements whose mineralogical form in the host rocks is the same as the ore minerals of interest, (e.g. W and Be), far fewer samples need be collected. An example would be that with an a priori probability of occurrence of 0.40, at a probability of recognition of 0.95, and wishing to obtain at least three samples again with values greater than the threshold, only 14 randomly collected samples are required. This latter example demonstrates the clear advantages of identifying appropriate pathfinder elements for deposit types whose major commodity is a ubitiquous element.

The technique would appear to have particular application to categorizing intrusive rocks or volcanic sequences such as found in Archean greenstone belts. Table 33.1 allows one to investigate the choice of increasing R (the minimum number of above threshold, T, individuals) or increasing the probability of recognition. If in the first example above R was

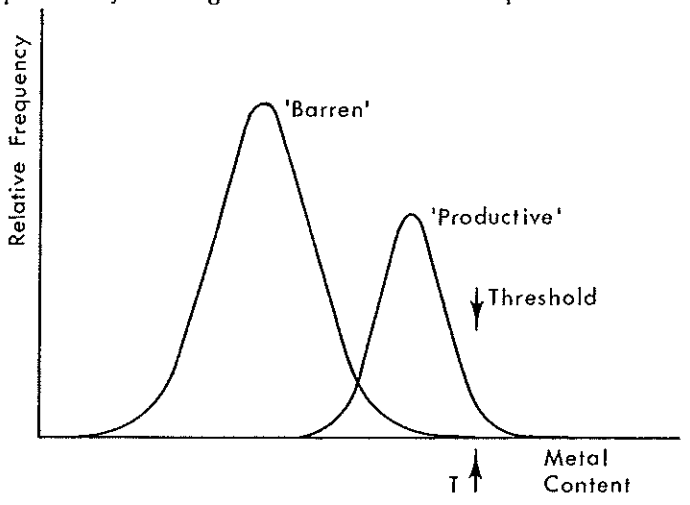

Figure 33.3. Example of histograms for 'barren' and 'productive' units.

increased to 4 a new total of 76 samples need to be collected and anlayzed for a decision at the 0.95 level. However, if R is held at 3 and the decision probability raised to 0.98, then 73 samples need to be acquired. In general, Table 33.1 reveals that it is less costly to increase the probability level associated with the decision than to increase the minimum number of above threshold samples. It should be noted that although the examples given above are for thresholds that are above the modes for the two distributions, the whole process can be reversed and the threshold be set to use very low values below the modes of the two distributions.

It is important to note that in both the latter two methods for sample size determination the samples from the unit should be selected randomly, i.e. giving all available parts of the unit an equal chance of being sampled. Collecting all the samples in one part of a unit and then using these to infer the properties of the whole is an invalid, and dangerous, procedure. A common approach is to grid the unit so that the appropriate number of samples may be collected, one from each grid cell or intersection. If this gridding technique is used and targets within the unit, rather than just associated with it, are expected, then the area of the target that would be intersected at some stated expectation can be estimated:

$$At = E(h).L^2.d$$

where At is the area of the target; E(h) is the expectation of a 'hit' (often set at 0.95); L is the side of the grid square; and d is the factor described previously (i.e. 1.08814). Once such an area has been estimated the linear form of the target can be approximated with a circle, ellipse or rectangle of equal area. However, this latter procedure is usually only of curiosity value as the targets may be buried within the unit or emplaced in other geological formations nearby.

Apart from the more general topic of coefficients of variation and their relationship to geochemical relief, the methods discussed in this section seem to be particularly applicable to lithogeochemical, soil/overburden and lake sediment surveys. Regional stream sediment surveys pose some different problems due to the formation of extensive

Table 33.1

Table of optimum sample sizes

OPTIMAL SAMPLE SIZES TO ENSURE THAT WITH STATED CONFIDENCE AT LEAST R ITEMS WILL EXCEED A VALUE T WITH PROBABILITY OF OCCURRENCE P

PROBABILITY OF RECOGNITION = .90
A PRIORI PROBABILITY OF OCCURRENCE, P

R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.12	.14	.16	.18	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90
1	230	114	76	57	45	38	32	28	25	22	19	16	14	12	11	9	7	6	5	4	4	3	2	2	2
2	388	194	129	96	77	64	55	48	42	38	31	27	23	21	18	15	12	10	9	8	7	5	4	4	3
3	531	265	176	132	105	88	75	65	58	52	43	37	32	28	25	20	16	14	12	10	9	7	6	5	4
4	667	333	221	166	132	110	94	82	73	65	54	46	40	36	32	25	21	18	15	13	12	9	8	6	5
5	798	398	265	198	158	132	113	98	87	78	65	56	48	43	38	30	25	21	18	16	14	11	9	8	7
6	926	462	308	230	184	153	131	114	101	91	76	65	56	50	45	35	29	25	21	19	17	13	11	9	8
7	1051	525	349	262	209	174	149	130	115	104	86	73	64	57	51	40	33	28	24	21	19	15	13	11	9
8	1175	587	390	292	234	194	166	145	129	116	96	82	72	63	57	45	37	32	27	24	21	17	14	12	10
9	1297	648	431	323	258	215	184	160	142	128	106	91	79	70	63	50	41	35	30	27	24	19	16	13	11
10	1418	708	471	353	282	235	201	175	156	140	116	99	87	77	69	55	45	38	33	29	26	21	18	15	13

PROBABILITY OF RECOGNITION = .95
A PRIORI PROBABILITY OF OCCURRENCE, P

R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.12	.14	.16	.18	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90
1	299	149	99	74	59	49	42	36	32	29	24	20	18	16	14	11	9	7	6	6	5	4	3	2	2
2	473	236	157	117	93	78	66	58	51	46	38	32	28	25	22	18	14	12	10	9	8	6	5	4	3
3	628	313	208	156	124	103	88	77	68	61	51	43	38	33	30	23	19	16	14	12	11	8	7	6	5
4	773	386	257	192	153	127	109	95	84	76	63	53	47	41	37	29	24	20	17	15	13	11	9	7	6
5	913	456	303	227	181	150	129	112	100	89	74	63	55	49	44	34	28	24	21	18	16	13	10	9	7
6	1049	523	348	261	208	173	148	129	115	103	85	73	63	56	50	40	33	28	24	21	18	15	12	10	8
7	1182	590	392	294	234	195	167	146	129	116	96	82	71	63	57	45	37	31	27	24	21	17	14	12	10
8	1312	655	436	326	260	217	185	162	143	129	107	91	79	70	63	50	41	35	30	26	23	19	16	13	11
9	1441	719	478	358	286	238	203	178	158	142	117	100	87	77	69	55	45	38	33	29	26	21	17	14	12
10	1568	782	521	390	311	259	221	193	172	154	128	109	95	84	76	60	49	42	36	32	28	23	19	16	13

PROBABILITY OF RECOGNITION = .98
A PRIORI PROBABILITY OF OCCURRENCE, P

R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.12	.14	.16	.18	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90
1	390	194	129	96	77	64	54	47	42	38	31	26	23	20	18	14	11	10	8	7	6	5	4	3	2
2	581	290	193	144	115	95	81	71	63	56	47	40	34	30	27	21	17	15	12	11	9	7	6	5	4
3	749	374	248	186	148	123	105	92	81	73	60	51	45	39	35	28	23	19	16	14	12	10	8	6	5
4	906	452	300	225	179	149	127	111	98	88	73	62	54	48	43	34	28	23	20	17	15	12	10	8	6
5	1055	526	350	262	209	174	148	129	115	103	85	73	63	56	50	39	32	27	23	20	18	14	12	9	8
6	1200	598	398	298	237	197	169	147	131	117	97	83	72	64	57	45	37	31	27	23	21	16	13	11	9
7	1340	669	445	333	265	221	189	165	146	131	109	93	81	71	64	50	41	35	30	26	23	19	15	12	10
8	1478	737	490	367	293	243	208	182	161	145	120	102	89	79	71	56	46	39	33	29	26	21	17	14	11
9	1614	805	535	401	320	266	227	199	176	158	131	112	97	86	77	61	50	42	36	32	28	23	19	15	13
10	1747	872	580	434	346	288	246	215	191	171	142	121	106	93	84	66	54	46	40	35	31	25	20	17	14

PROBABILITY OF RECOGNITION = .99
A PRIORI PROBABILITY OF OCCURRENCE, P

R	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.12	.14	.16	.18	.20	.25	.30	.35	.40	.45	.50	.60	.70	.80	.90
1	459	228	152	113	90	75	64	56	49	44	37	31	27	24	21	17	13	11	10	8	7	6	4	3	2
2	662	330	219	164	130	108	92	81	71	64	53	45	39	34	31	24	20	16	14	12	11	8	7	5	4
3	838	418	277	207	165	137	117	102	91	81	67	57	50	44	39	31	25	21	18	16	14	11	9	7	5
4	1001	499	332	248	198	164	140	122	109	97	81	69	60	53	47	37	30	25	22	19	17	13	11	9	7
5	1157	577	383	287	229	190	162	142	126	113	93	79	69	61	55	43	35	30	25	22	19	15	12	10	8
6	1307	652	433	324	259	215	184	160	142	127	106	90	78	69	62	49	40	34	29	25	22	18	14	12	9
7	1453	725	482	360	288	239	204	178	158	142	118	100	87	77	69	54	44	37	32	28	25	20	16	13	11
8	1596	796	529	396	316	263	225	196	174	156	129	110	96	85	76	60	49	41	36	31	27	22	18	15	12
9	1736	866	576	431	344	286	244	213	189	170	141	120	104	92	83	65	53	45	39	34	30	24	20	16	13
10	1874	935	622	465	371	309	264	230	204	183	152	130	113	100	89	70	58	49	42	37	33	26	21	18	14

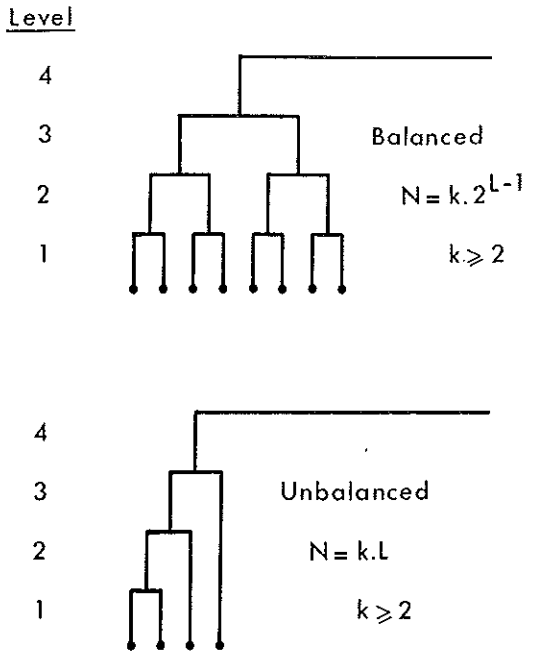

Figure 33.4. Heirarchical sample designs.

linear dispersion trains. Studies are currently underway using the techniques of autocovariance to investigate these dispersion trains and to determine the 'memory' of stream sediments in different physical environments and for different elements. A more formal determination of dispersion train geometry may eventually lead to numerical decision criteria for selecting sample spacings along stream drainages.

Evaluation Mode Considerations

In 'evaluation' mode surveys the ultimate objective is to produce a stable map which can be used to study systematic compositional changes. By stable is meant a map where local variability due to sampling and analytical sources is significantly smaller than variation across the area of interest. If the map is unstable, i.e. local variability is as large, or larger than, regional variability, systematic compositional changes will be unrecognizable. The classical method, or model, used to investigate such problems is analysis of variance (ANOVA), and the early applications in geology by Krumbein and Slack (1956) still stand as a landmark. Since that time Miesch and his coworkers in the Branch of Regional Geochemistry of the U.S. Geological Survey, Denver, have done much to develop the use of ANOVA methods in geochemical studies (e.g. Miesch, 1976). One of the major drawbacks of classical ANOVA studies has been the very large sampling and analytical load generated by multilevel studies of variability. In the past, except for some work at the U.S. Geological Survey, Denver, and at the Geological Survey in Ottawa, work has been largely restricted to the use of what are known as balanced designs (Fig. 33.4). In balanced sampling designs, samples are collected in equal replicates (usually pairs) at each level of the investigation. The notation used in Figure 33.4 is as follows: N is the total number of samples to be collected and analyzed, these consist of k replication groups each with L sampling levels of a nested, or hierarchical, layout. The number of replicated groups, k, has to be greater than two in order to assess the variability at the highest level. If the balanced design in Figure 33.4 described a bedrock study, then level 1 might represent analytical variation, level 2 variation in the outcrop, level 3 variation between outcrops a 100 m apart, and level 4 variation between groups of outcrops across the geological unit. Many package computer programs exist to handle such designs, but it can be seen that as the number of levels of variability increase, the number of samples required increases exponentially. In the four level design using pairs, multiples of 8 samples are required to study variability across a geological unit. Unbalanced designs are far more efficient in their use of survey resources, and in the four level example only multiples of 4 samples are required. The impact of using balanced (B) and unbalanced (UB) designs can be seen in Figure 33.5. The Y axis, N/K, of the figure is the number of samples needed for each replication of simple balanced (B) or unbalanced (UB) designs with the number of levels in the design increasing from 2 to 6. The relative efficiency of balanced versus unbalanced design can be obtained from the ratio UB/B and the right hand Y axis. For instance with a four level design the ratio is 0.5 and an unbalanced design requires only half as many samples as a balanced one. Unbalanced designs are particularly advantageous where greater than four levels are used and savings in sampling and analytical costs of 50 per cent and higher can be made. The problem in the past has been that the potential of unbalanced designs has not been publicized, and, perhaps more importantly, the more complex computer programs necessary to carry out unbalanced ANOVA studies have not been widely available. A computer program, UANOVA, to carry out the computations has been written in cooperation with Systems Approach Consultants Ltd. of Ottawa and will be published (Garrett and Goss, in prep.). It is hoped that the availability of this software to handle complex unbalanced data structures will encourage a greater use of ANOVA techniques in a variety of geological studies.

An application of unbalanced designs to regional reconnaissance surveys has been presented by Garrett and Goss (in press) and facets of the underlying statistical ANOVA model

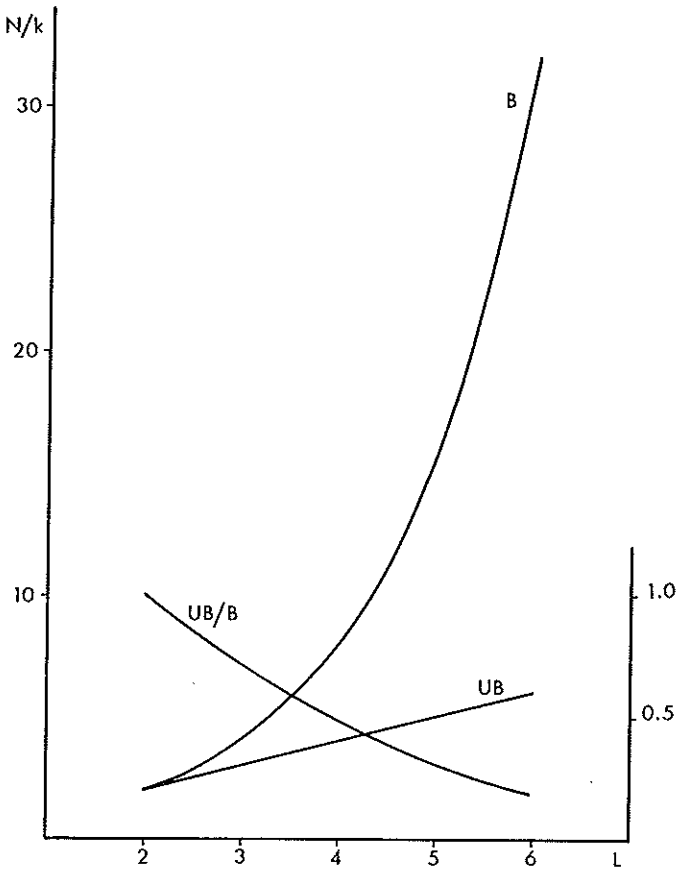

Figure 33.5. Relationship of sample requirements to number of levels.

Table 33.2

Analysis of variance to determine significance of variance components for uranium in the northwestern Ontario area

Variation Between	Sum of Squares	df	Mean Squares	Unit Size	Variance Component	V.C. %	Error Mean Squares	Error df	Approx F	Signif
Cells	320.73478	1679	0.19103	1680	0.101481	61.05	0.07054[1]	108.47[1]	2.71	>.999
Lakes in Cells	9.73124	105	0.09268	1785	0.059743	35.94	0.00525[1]	118.55[1]	17.67	>.999
Samples in Lakes	0.57682	105	0.00549	1890	0.001907	1.15	0.00309	105.00	1.78	.998
Analyses in Samples	0.32447	105	0.00309	1995	0.003090	1.86				
Total	331.36731	1994			0.166221	100.00				

[1]Error mean square synthesized and degrees of freedom computed by Satterthwaites (1946) formula

and assumptions by Goss and Garrett (in press). In 1977 the sampling design of the National Geochemical Reconnaissance lake sediment surveys carried out on the Canadian Shield was modified so that the proportion of the variation at regional, 13 km^2 grid cell, within lake and analytical levels could be determined. The design was essentially similar to that used as an example for the unbalanced case in Figure 33.4. The results of data for uranium in the 1977 Ontario survey are presented in Table 33.2. The distribution of the variability reveals some 61 per cent of the variation to be between the 13 km^2 cells, (i.e. regional), 36 per cent is due to lakes in cells, and the remaining 3 per cent is due to the effects of within lake sampling and analysis. These results are not too different in trend from those obtained for all 1977 lake sediment surveys where it was observed that some 65 per cent of the uranium variation was at the regional level, 30 per cent at the 13 km^2 level and the remaining 5 per cent at the within lake and analytical level. From the work so far carried out in exploration geochemistry it is clear that analytical variability is a very small contributor to the total variation in all but those trace elements, such as As, Hg and Ag, whose analysis at abundance levels is most difficult.

The old balanced designs lead to an abundance of anlayses at the lowest level where, in general, there is only a small portion of the total variability. This is wasteful of resources and it would be more useful to have larger numbers of samples analyzed from levels higher in the design so increasing the statistical degrees of freedom at those levels. The unbalanced design has just this effect and it can be seen in Table 33.2 that for the lowest three levels the degrees of freedom (df) are all equal, in fact corresponding to the number of blocks (replications) of data used in the example. The example in Table 33.2 also demonstrates a type of unbalanced layout, known as an inverted design (Bainbridge, 1963). In balanced ANOVA designs there are always more degrees of freedom at the bottom of the design, usually where they are not needed. In an inverted design there are more degrees of freedom at the top where they are most needed. In a regional survey the degrees of freedom at the top of the inverted design are mostly due to the samples collected singly from grid cells without any replication.

Apart from the establishment of the relative variability in the data, the variance components for each level can be tested to determine if they are significantly different from zero. If they are not significantly different from zero, then it is probable that the variation at that particular level can be explained by variation at lower levels. In the Table 33.2 example it can be seen that variation at the level of samples in lakes (level 2) is small and in part is due to the analytical variation. Following similar findings in the other areas where lake sediment surveys were carried out in 1977 for the National Geochemical Reconnaissance replication at this level (2) will probably be discontinued after 1978. At the level of variation between lakes within 13 km^2 grid cells (level 3) there is significant variability relative to the within lake and analytical components. From this it may be inferred that the sample density could be increased, if so desired, and the resolution of the geochemical map improved as there are mappable geochemical features at finer than 13 km^2 scale. Again this is not an unexpected situation in the relatively geologically heterogenous area of Shield rocks of the survey area. Finally, at the regional level, the variability is highly significant in terms of the lower local variability. This knowledge can only give one confidence in the geochemical map resulting from the survey and make one confident that the regional variations are real geochemical phenomena and not the result of random variation at local and analytical levels.

Traditionally, ANOVA techniques found their greatest application in orientation and sampling design studies, not in actual surveys, due to the very large overhead imposed by use of classical balanced designs. With the introduction of unbalanced designs, it has become feasible to use the ANOVA model as a quality control tool to continuously monitor relative variability and significance in large regional geochemical surveys. The main criteria for selection of sample densities in these surveys is a knowledge of the size of the dispersion targets being sought and the availability of funds. The information gained from using the ANOVA model as a quality control tool will allow modifications to be made to the basic sampling design evolved from orientation surveys. This can be particularly important if orientation survey data are expanded in application to cover geological and physiographic environments somewhat dissimilar to the original orientation area. Nothing can usually be done to put right a survey that has gone wrong, but the quality control monitoring should at least prevent making the same mistakes a second time.

The Combined OR-ANOVA Model

The techniques described for selecting grid size and the selection of minimum sample sizes via binomial probability may be considered Operations Research (OR) methods. They allow the planning of apparently optimal sample designs after orientation studies and before major surveys. In all likelihood the actual survey will not work out exactly as predicted and it would be beneficial to be able to assess the success of the survey after it has been carried out.

The combined OR-ANOVA model is an attempt to make use of the pre- and post-survey information and combine it to create an a posteriori probability of success for finding a particular target. The purely geometric aspect of estimating the probability of a sample falling within a dispersion halo is

combined with an empirical ratio derived from an unbalanced ANOVA model, which provides an a posteriori estimate of the probability of recognition. The product of these two probabilities will give an estimate of the a posteriori probability of success of a survey. Interest in this parameter is not purely academic. Areas for regional geochemical reconnaissance are commonly chosen because of geological indications that significant mineral occurrences are present. After a survey has been executed, and no promising results obtained, the question arises: is the area devoid of significant mineral occurrences detectable by the particular geochemical method, or was the survey technically insufficient? If the a posteriori probability of success was high and the survey well designed, then one would have to accept the hypothesis that it is unlikely, at some stated confidence level, that detectable significant mineral occurrences exist, and, it would be a poor decision to continue certain kinds of work in the area. Or alternatively, that the original assumptions as to the mineral potential of the area were wrong. Conversely, if the a posteriori probability was low, then it would have to be accepted that one way or another the survey was poor and should not be used alone, or at all, to write off an area as being unworthy of further work.

The combined OR-ANOVA model is put into the context of national regional geochemical surveys in Figure 33.6. These surveys are commonly multielement but 'tuned' for a particular element in a particular environment, and, as a result, they will be inadequate for some elements and overkill for others. It could be argued that the decision tree shown is idealistic in the sense that the maps or data will be published

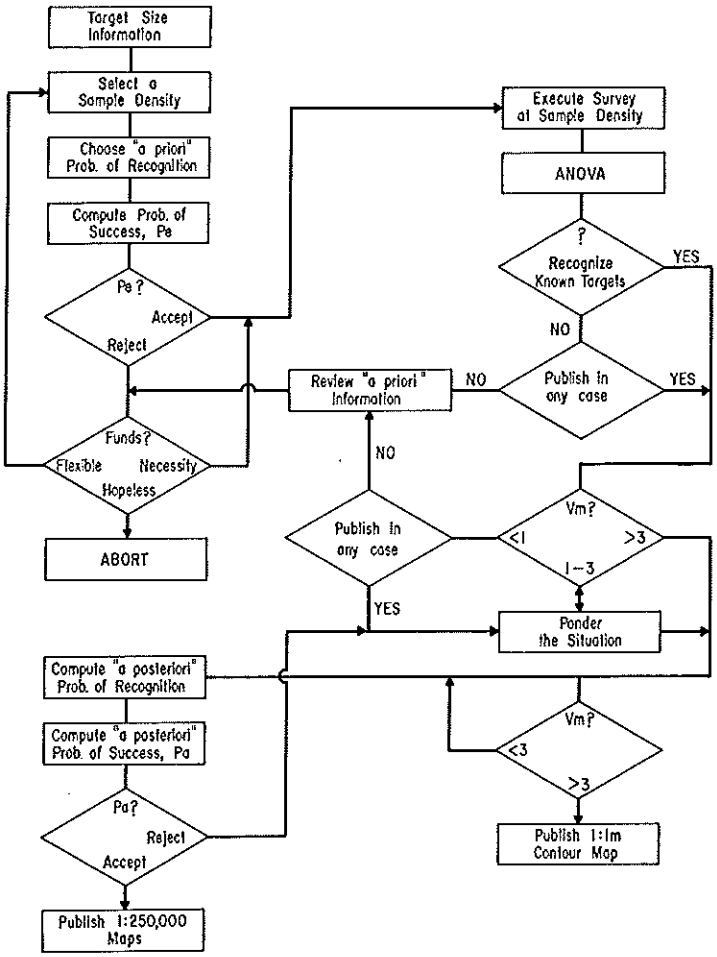

Figure 33.6. The combined OR-ANOVA model.

in any case. However, as unflattering as the tree is in places, it is probably close to reality. Even though a survey may have an associated probability of success that is low, this does not invalidate the map as a prospecting tool. Anomalous, above threshold, patterns should lead to mineral occurrences with correct follow-up procedures. In that sense, the survey can be deemed a success; however, in the absence of those successes, it would be extremely unwise to write the area off purely on the basis of such a geochemical survey.

Two numerical steps in Figure 33.6 need additional comment. A parameter V_m is introduced as being important, particularly in the decision as whether to publish 1:1 million scale contour maps. V_m is a measure of map stability proposed by Miesch (1976). This empirical variance ratio is derived from the ANOVA model and has proved to be a good practical guide to reliable (i.e. reproducible) contour map construction. Experience has shown that surveys yielding V_m ratios in excess of 3 lead to stable maps. Where lower values of V_m are found it is unsound policy to publish contour maps as replicate surveys for the area would very probably yield significantly different maps. Additionally, in such a case it would be unwise to continue with any sophisticated data analysis or production of derivative maps. In certain cases maps of greater stability, but lower resolution, can be prepared by using the means of blocks of adjacent data (Miesch, 1976). A low V_m would not invalidate some other kind of map at survey scale, say 1:250 000, using a symbolic or numerical presentation; however, one would be less confident in making geochemical or geological inferences as to the true patterns of regional variation from those maps.

The second numerical step needing additional comment is the flow from the ANOVA results to an a posteriori probability of recognition. This step is the subject of current research and the following method is only tentative. Empirical variance ratios are excellent guides to relative variability and are easily derived from the variance components computed in the ANOVA. Using the lake sediment survey case (Table 33.2) as an example, two ratios, V_1 and V_2, can be calculated that have some geochemical meaning. Let v_1, v_2, v_3, and v_4 respectively be the components of variance for the analytical, within lake, within cell and regional levels; and v_{123} be the sum of the of the components from the analytical up to the within cell level, and v_{12} be the sum of up to the within lake level, then

$$V_1 = v_4/v_{123}$$

and

$$V_2 = v_3/v_{12}$$

If V_1 is high, then most of the variation is regional, between cells, rather than local within cells, and a reliable map is derived from the data, in general values above 1 are at least adequate. Even if V_1 is low, the map is not worthless from a prospecting viewpoint. V_2 is a measure of the reliability within a cell, i.e. how good is the map at revealing local variation patterns. Again with V_2, as with V_1, one would hope to obtain high values, inferring that the variability due to sampling in a single lake, and analysis, is small. These two ratios, V_1 and V_2, intuitively seem to be useful as a guide to the probability of recognition, P(r), and could be used in an appropriate function, f,

$$P(r) = f(V_1, V_2)$$

However, the empirical variance ratios, V terms, are somewhat inconvenient as they range from zero to infinity. A convenient transform can be used, the inverse Fisher, to convert them to the range zero to one. Let x be the appropriate value of V_1 or V_2 and Z_1 or Z_2 be the corresponding transformed values, then

$$Z = (e^X-1)/(e^X+1)$$

Now the previous function can be rewritten

$$P(r) = f(Z_1, Z_2)$$

and two obvious candidates for explicit calculation of the function are

$$P(r) = Z_1.Z_2$$

and

$$P(r_2) = a_1Z_1+a_2Z_2$$

where a_1+a_2 are weights and sum to one. In both explicit functions P(r) will take on a value of between zero and one. In the case of the test data in Table 33.2, the corresponding values of P(r) are 0.65, and 0.83 if the weights a_1 and a_2 are equal, i.e. both 0.5 (Table 33.3). In terms of prospecting, local reliability may well be more important in leading to a mineral occurrence than broad scale regional reliability. If a_1 and a_2 reflect this by local reliability being thrice as important, i.e. $a_1 = 0.25$ and $a_2 = 0.75$, then a value for P(r) of 0.91 is obtained. If this result is applied to the demonstration case in Figure 33.1, i.e. the expectation of a 'hit' on a target, E(h), 6 units long by 1.2 units wide is 0.95, the actual expectation that the 'hit' will be recognized as such is the product of E(h) and $P(r_2)$, i.e. 0.86, which is approximately 9 times out of 10. Note that regardless of the target size, E(h) can never exceed 1.0; therefore, the expectation of recognition can never exceed P(r), which in this example is 0.91. This intuitively seems to be an underestimate, for if there were a cluster of sample sites within the target some at least would be recognized. This topic remains to be investigated in greater depth.

Practical experience with the combined OR-ANOVA model will have to reveal its true value. The most tenuous step at the moment is in the derivation of the a posteriori probability of recognition. The procedure outlined above is simplistic and direct. An alternative method, yet to be investigated, is one using Bayesian probability, which may prove to be the better method. It is planned to investigate the 1977 and 1978 National Geochemical Reconnaissance data using the OR-ANOVA model; this should shed light both on the reliability of the maps and data, and the model. The evaluation will have to be empirical as the truth as to the actual mineral resource endowments of the survey areas is not known.

Table 33.3

Generation of P(r)

$v_4 = 0.101481$

$v_3 = 0.059743$

$v_{123} = 0.064740$

$v_2 = 0.001907$

$v_{12} = 0.004997$

$v_1 = 0.003090$

$V_1 = v_4/v_{123} = 1.5675$ and $Z_1 = 0.6549$

$V_2 = v_3/v_{12} = 11.9558$ and $Z_2 = 1.0000$

$P(r_1) = Z_1.Z_2$

$R(r_2) = a_1Z_1+a_2Z_2$

Therefore $P(r_1) = 0.65$

and $P(r_2) = 0.83$ with $a_1 = a_2 = 0.5$

and $P(r_2) = 0.91$ with $a_1 = 0.25$ and $a_2 = 0.75$

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