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A COMPUTER PROGRAM FOR FACTOR
ANALYSIS OF GEOCHEMICAL AND
OTHER DATA

E. M. Cameron

DEPARTMENT OF ENERGY, MINES AND RESOURCES

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ABSTRACT

A program, written in Fortran II for a CDC 3200 computer with a 16K memory, is given for the factor analysis of data containing up to 36 variables. Principal components are extracted by the Jacobi method and up to 26 factors are rotated first to orthogonal simple structure using the Varimax criterion and then to oblique simple structure by the Promax method. Factor scores are computed by Harmon's (1960) method of ideal variables. A separate program computes scores from either orthogonal or oblique factor matrices by a method of Kaiser's (1962).

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A COMPUTER PROGRAM FOR FACTOR ANALYSIS OF GEOCHEMICAL AND OTHER DATA

INTRODUCTION

In the first decade of this century Pearson and then Spearman presented the essential concepts of factor analysis as a method for resolving the intercorrelations between variables. Since that time the method has become an important tool for interpreting complex psychological tests. The tedious computations involved in factor analysis, however, retarded the full flowering of applications of the method until the digital computer became widely available. One of the sciences most likely to benefit from the application of factor analysis is geology, for the geologist, like the psychologist, is frequently concerned with deducing underlying controls or processes from observations.

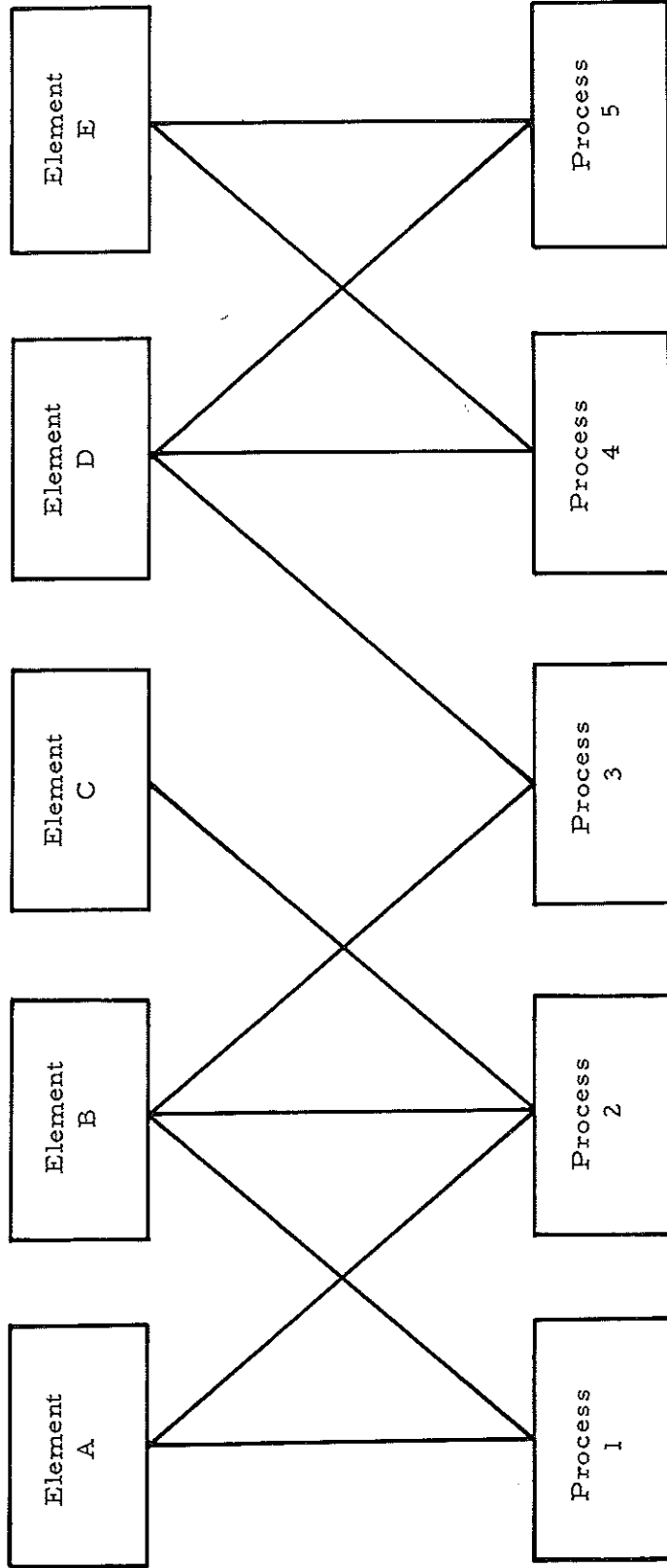
The introduction of factor analysis techniques to geology has fortunately coincided with the development of very rapid instrumental methods of geochemical analysis (e.g. Cameron and Horton, 1967). The large amounts of data which these methods produce take a long time to interpret by conventional techniques. Factor analysis and other statistical methods go a long way towards eliminating the tedious portions of geochemical interpretation, leaving the geologist or geochemist to interpret relatively small and simple data matrices.

It is not appropriate here to detail the principles of factor analysis. Harmon (1960) has written an exhaustive text and Cattell (1965a, 1965b) gives an excellent review of the subject. Imbrie (1963) and Spencer (1966) present summaries and report respectively, geological and geochemical applications. The utility of factor analysis when applied to geochemistry can be appreciated if it is considered that the distribution of each element in a rock unit is more frequently than not determined by a number of different geological processes; conversely, each process usually affects a number of elements. These relationships are shown in Figure 1. Thus if the distribution of an element within a geological system is studied, the geochemist frequently finds himself considering a complex distribution that is, in fact, the sum of a number of distributions derived from different geological processes. It is more rewarding instead to consider the effects of processes within the system. Factor analysis allows the investigator to move from the case-variable level of his data to the population-process level.

The work of Imbrie and his colleagues (Imbrie, 1963; Imbrie and Purdy, 1962; Imbrie and Van Andel, 1964; Manson and Imbrie, 1964; Klován, 1966) has played an important part in introducing factor analysis to the

Manuscript received April 25, 1967.

CASE - VARIABLE LEVEL OF DATA



POPULATION - PROCESS LEVEL OF DATA

Figure 1. Relationships between element distribution and geological processes.

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geologist. Imbrie has particularly emphasized the use of Q-mode methods of analysis where a matrix of correlations between the samples is factored, rather than a matrix of correlations between variables as in R-mode methods. A feature of Q-mode analysis that is of immediate appeal to the geologist is that the factor loadings are a measure of the effect of the different factors on each sample. These loadings can, therefore, be plotted directly onto a map or section for further interpretation. The disadvantage of Q-mode methods for geochemical studies arises from the need to factor a matrix of the same order as the number of samples. Because the time taken to extract factors by the Jacobi method is approximately proportional to the cube of the order of the matrix, Q-mode analysis consumes more computer time than R-mode analysis when the number of samples is greater than the number of variables. More importantly, factoring a moderately large sample matrix may be beyond the memory capacity of the available computer (matrices up to order 36 can be factored with the 16K memory of the CDC 3200 using GEOFACT). For most geochemical studies it is therefore more practical to use an R-mode method and then obtain factor measurements on each sample by computing factor scores.

In writing a program for the factor analysis of geochemical data a major consideration was that the program be readily convertible for use on other computer installations. Thus the program has been made as short and simple as possible, consistent with computing efficiency. The CDC 3200 compiler accepts many of the elements of the Fortran IV language, but an attempt has been made to use only those elements that are commonly found in both Fortran II and IV compilers. In the interests of simplicity the number of program options has been kept to a minimum; the optional input of matrices to any part of the program allows a considerable degree of flexibility in modifying the techniques. A maximum of 36 variables may be factored and up to 26 factors rotated to orthogonal and oblique simple structure by GEOFACT. These limits are dictated only by the size of the core memory of the CDC 3200 computer that is used. This is 16K machine words, one of which is used per integer number and two per real number. Each real number has 11 digits precision.

ACKNOWLEDGMENTS

Much of the running and testing of this program has been done by Mr. Ken Stewart. The cooperation of Dr. K.W. Dawson, Mr. L.H. Boutet and Miss S.M. Cooper expedited the punching and running of the program. I am grateful to Dr. F.P. Agterberg for drawing to my attention the Steeples and Lohnes coding of the Varimax method of rotation and to Mr. D.W. Ponder for critically examining the manuscript. Finally, my gratitude to Dr. Derek Spencer for many hours discussing the application of factor analysis.

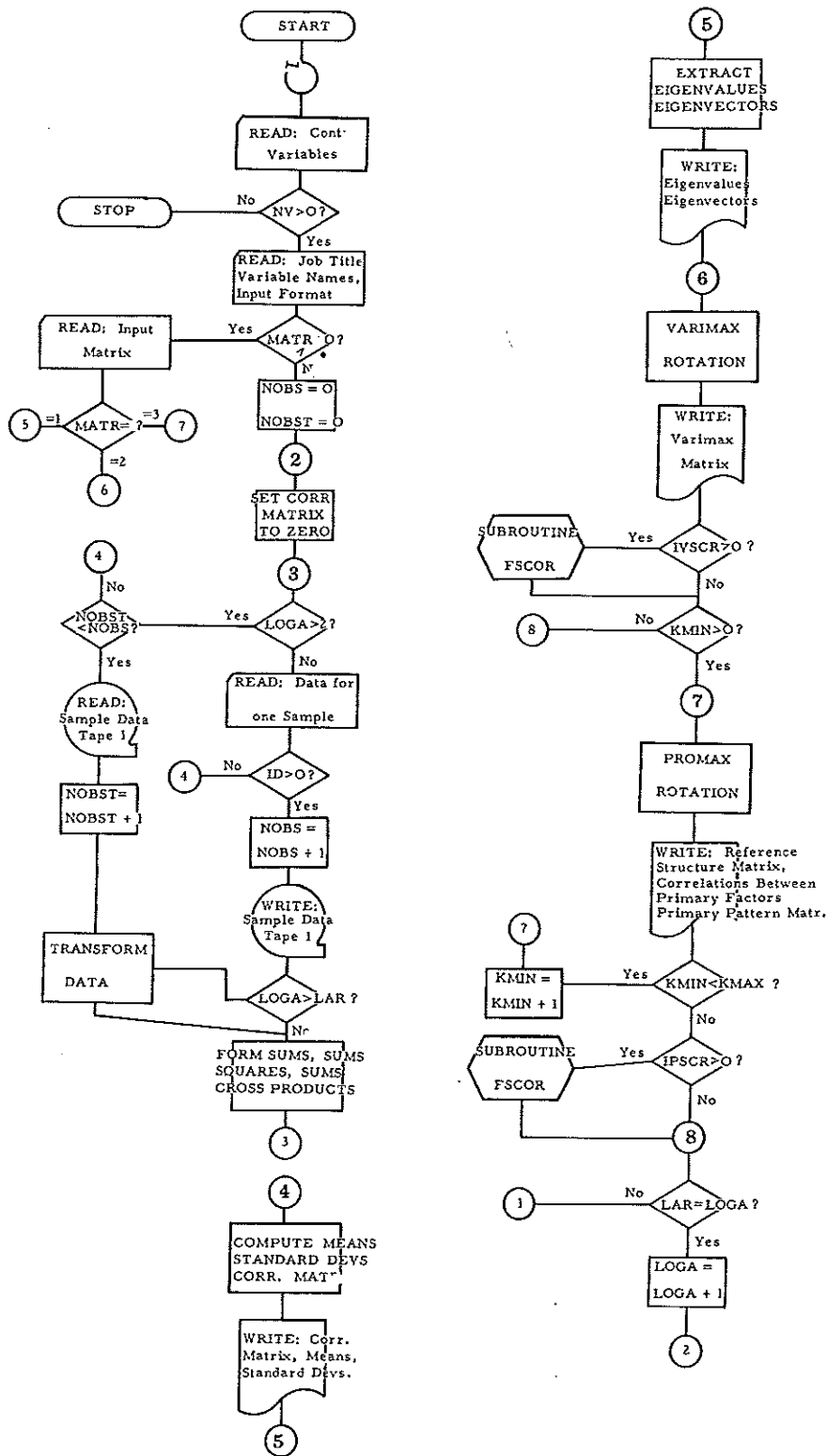


Figure 2. Skeleton flow chart for Program GEOFACT.

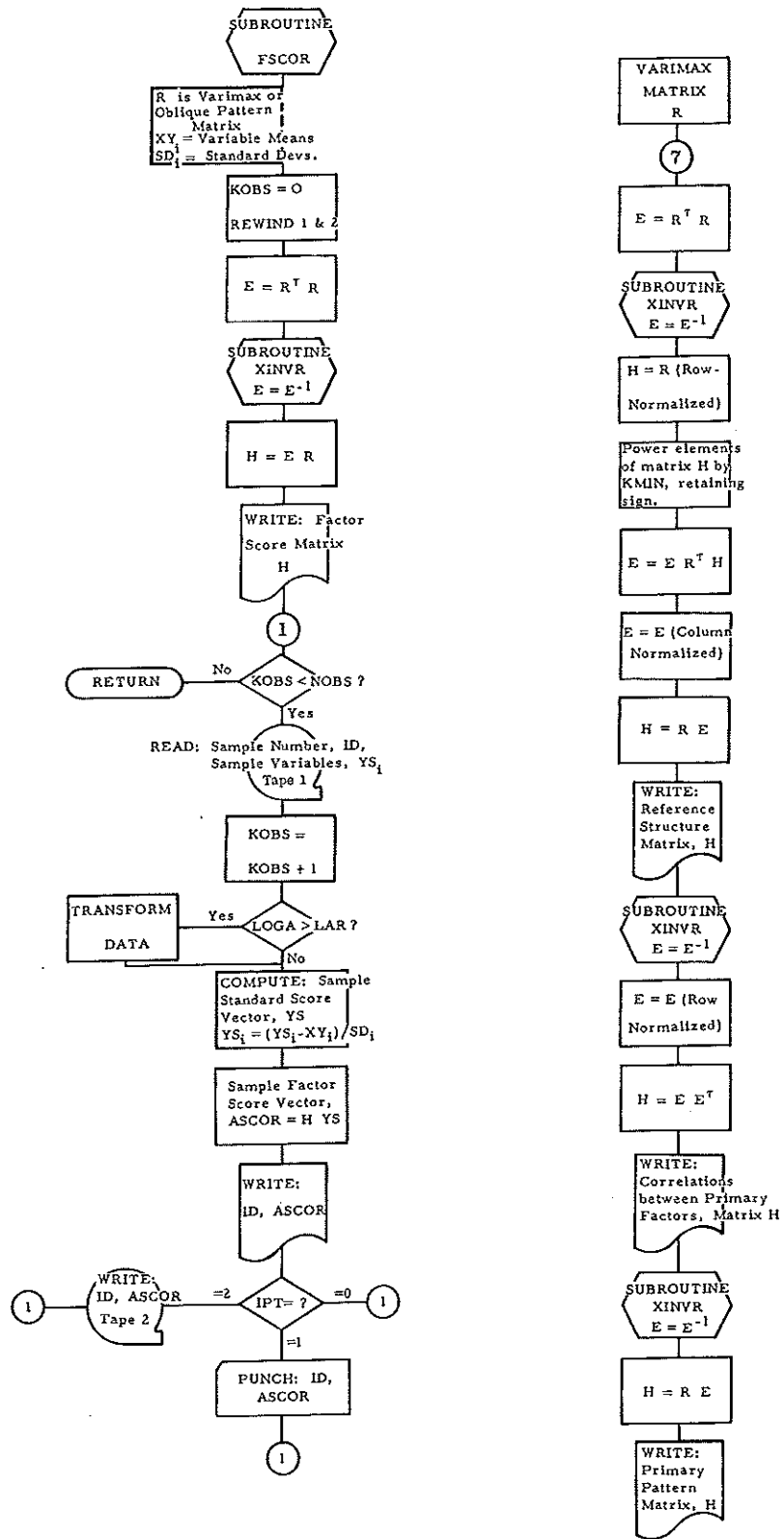


Figure 3. Flow chart for Subroutine FSCOR (left) and for the Promax Rotation segment of GEOFACT (right).

PROGRAM DESCRIPTION

Statements 1-99, data input and correlation matrix.

The Pearson product-moment correlation matrix is computed together with variable means and standard deviations. Because many geochemical data approximate a lognormal distribution, an option allows for the data to be transformed to logarithms prior to any computation. If another type of transformation is preferred, statements 12 and 13 may be replaced by appropriate statements.

Statements 100-199, extraction of principal components matrix.

The latent roots and vectors of the correlation matrix are extracted by the Jacobi method. This segment of the program is a modification of subroutine HDIAG (Cooley and Lohnes, 1962). An account of the method, with a flow chart, is given by Greenstadt (1960). The correlation matrix that is treated has unities in its principal diagonal. Strictly speaking, therefore, GEOFACT performs component analysis rather than factor analysis (Cattell, 1965a). The program can, however, be readily modified to compute and insert communalities in the diagonal if this is desired; alternatively, without modifying the program, a matrix with communalities may be entered for factoring using the matrix entry option. There have been too few factor analysis treatments of geochemical data to properly evaluate the usefulness of inserting communalities rather than unities. Harmon (1960) and Cattell (1965a) discuss this widely debated problem in some detail.

Statements 200-299, orthogonal rotation.

The principal components matrix is rotated to orthogonal simple structure using Kaiser's (1958) Varimax criterion. This segment of the program has been modified from a program coded by T. C. Steeples and modified by P. R. Lohnes of the University of New Hampshire. The principal components matrix is row-normalized for the specified number of factors prior to rotation, then denormalized after rotation. Rotation of the matrix is terminated when the angle of further rotation for all pairs of factors is reduced to a trivial level.

The number of factors that are rotated is specified on the control card or alternatively, using the EV option, may be determined as being equal to the number of principal components with eigenvalues greater than a chosen value. Kaiser (1960) suggests that factors with eigenvalues less than 1.0 are generally not significant. It has been the writer's experience, however, that many sets of geochemical data contain geologically significant factors with

eigenvalues much less than 1.0. With geochemical data it has been found convenient to rotate all factors with eigenvalues greater than 0.1, and then on a second run drop off all those factors that do not appear to be significant.

Statements 300-399, oblique rotation.

It is rather unlikely in nature to find factors within a set of data that are independent of one another - an assumption that is made when rotating to an orthogonal solution. An efficient method for rotation to oblique simple structure is therefore desirable. The Biquartimin (Carroll, 1957) and Binormamin (Kaiser and Dickman, 1959) criteria are generally considered to give the most acceptable solutions. The writer has modified a program of Carroll's using the former criterion for the CDC 3200 and found it to give good results with geochemical data. However, it is a lengthy program and consequently the Promax method (Hendrickson and White, 1964) was investigated as an alternative for inclusion in GEOFACT. The Promax method can be programmed with a much smaller number of statements and, because it is not an iterative technique, consumes much less computer time than methods using either of the above two criteria.

In the Promax method the Varimax matrix is used to construct an "ideal" oblique solution; the Varimax matrix is then rotated to a least squares fit of this ideal solution. The "ideal" solution is obtained by row-column normalizing the Varimax matrix A to give matrix B, the elements of which are then powered by k, retaining their sign, to give matrix E:

$$e_{ij} = |b_{ij}^k|$$

Powering the elements of the Varimax matrix has the effect of proportionately increasing high loadings relative to low loadings. The least squares fit of A to E is found by using the Procrustes equation of Hurley and Cattell (1962):

$$C = (A^T A)^{-1} A^T E$$

After column normalization, C becomes the transformation matrix Y between the Varimax matrix and the oblique reference structure matrix V:

$$V = AY$$

By computing the inverse of Y and then normalizing its rows, the primary factor transformation matrix T^T is obtained. From this a matrix L of correlations between the oblique primary factors and the oblique primary pattern matrix P may be computed:

$$L = T^T T$$

and $P = A(T^T)^{-1}$

The elements of B may be powered by different values of k - Hendrickson and White (1962) suggest values of 2 to 4 - to obtain solutions that differ in their degree of obliqueness. The smaller the value of k, the weaker are the correlations between the oblique primary factors. The Promax method is so fast that, at least for the first run of a set of data, several rotations using different values of k should be made, setting KMIN and KMAX appropriately. Then the solution that makes most sense geologically can be chosen. The Swan Hills carbonates test data are cleanly structured and are most suitably powered by a k value of 2; more complexly intercorrelated geochemical data are best treated by higher values of k.

Subroutine FSCOR

Factor measurements by Harmon's method of "ideal variables" are computed thus:

$$f = (A^T A)^{-1} A^T Z$$

where f is a vector of sample factor scores, A is a factor pattern matrix and Z is a vector of sample standard scores. Because only one matrix is used in computing the factor scores, this method can very conveniently be included in the main program. A print-out of these scores will often be found useful for interpreting the factor matrices.

Unfortunately, the factor measurements given by this method are only approximate when the number of factors are less than the number of variables. Thus once the number of factors for rotation has been decided and the required rotated matrix computed, true factor scores may be found using the more complex formula employed by GEOSCORE.

Subroutine XINVR

This subroutine for matrix inversion follows the method described by Anon.(1961). It was adapted from a program coded by R. J. Bolton and G. Cameron of the Computer Science Division, Department of Energy, Mines and Resources.

Program GEOSCORE

For factors that have been obtained from a correlation matrix with unities in the diagonal (strictly, components derived by component analysis), the following formula from Kaiser (1962) may be used to compute factor (component) scores:

$$f = LA^T R^{-1} Z$$

where f is a vector of sample factor (component) scores, L is a matrix of correlations between oblique primary factors, A is the primary pattern matrix, R is the correlation matrix and Z is a vector of sample standard scores. When A is a matrix of orthogonal factors, L is an identity matrix and therefore need not be entered. Unlike the method of "ideal variables", this method gives true factor (component) measurements when the number of factors is less than the number of variables. If the correlation matrix was obtained from transformed data, the means and standard deviations of the transformed data should be entered to compute the factor scores. A flow chart for this program is given in Figure 4.

Suggested Procedure

It is suggested that the first run of a set of geochemical data be made using both the LAR and LOGA options in order that both the raw and the transformed data are factored; with the EV option set at 0.1; KMIN and KMAX at 2 and 4 respectively; and with IVSCR set at 1 to compute factor scores from the Varimax matrix by Subroutine FSCOR. Inspection of the output will show whether a transformation is necessary, the proper number of factors to retain and the most suitable value to power the elements of the matrix in the Promax rotation. The appropriate unrotated factor matrix punched out on the first run may then be entered (MATR = 2) and the chosen number of factors rotated to orthogonal and oblique simple structure. On this second run IOP is set to 1 in order that all matrices are punched out for use, if required, as input for GEOSCORE.

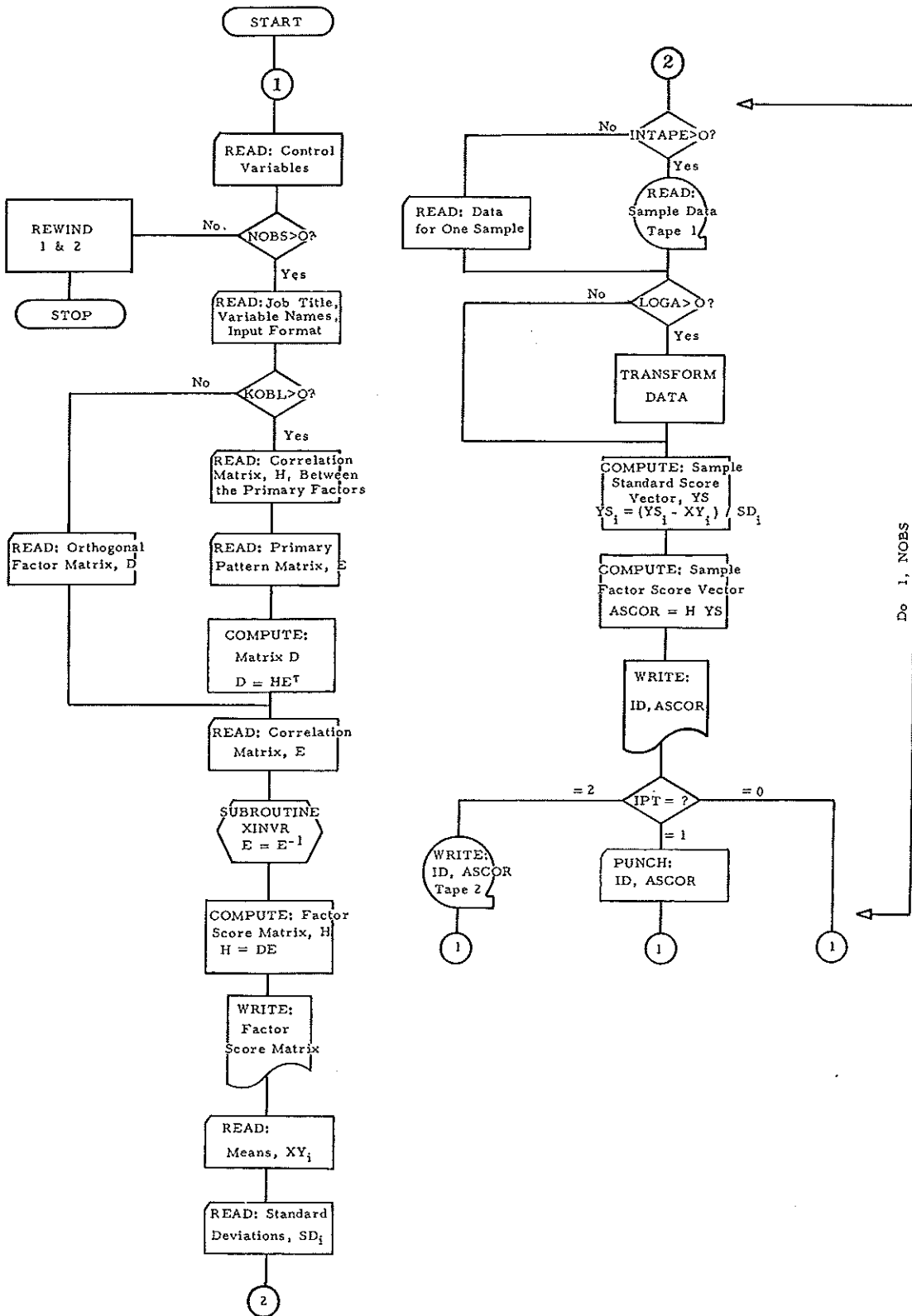


Figure 4. Flow chart for GEOSCORE.

Program listing for GEOFACT

```

PROGRAM GEOFACT
PROGRAMMED BY E M CAMERON GEOLOGICAL SURVEY OF CANADA FEB 1967
PROGRAMMED FOR CDC 3200 WITH 16K MEMORY,MAXIMUM OF TWO TAPE DRIVES REQUIRED
(TAPE 2 IS USED ONLY FOR OPTIONAL OUTPUT OF FACTOR SCORES)
PROGRAMMING LANGUAGE *FORTRAN II*
PROGRAM FOR FACTOR ANALYSIS OF GEOCHEMICAL AND OTHER DATA. INPUT IS RAW
ANALYTICAL DATA WHICH MAY BE OPTIONALLY TRANSFORMED TO LOGARITHMS. PROGRAM
COMPUTES CORRELATION MATRIX FROM WHICH THE PRINCIPAL FACTORS AND THEIR
EIGENVALUES ARE EXTRACTED. THE PRINCIPAL FACTOR MATRIX IS ROTATED TO
ORTHOGONAL SIMPLE STRUCTURE BY THE VARIMAX METHOD AND THEN TO OBLIQUE SIMPLE
STRUCTURE BY THE PROMAX METHOD. FACTOR SCORES MAY BE COMPUTED FROM EITHER
THE VARIMAX OR PROMAX MATRICES. MATRICES CAN BE ENTERED FOR FURTHER TREATMENT
A MAXIMUM OF 36 VARIABLES MAY BE ENTERED AND UP TO 26 FACTORS ROTATED
*****CARD INPUT AS FOLLOWS*****
CARD 1**CONTROL CARD**
COL.1-2*NV*NUMBER OF VARIABLES
COL.3-4*NF*NUMBER OF FACTORS FOR ROTATION
COL.6 *LAR**PUNCH 1 IF RAW DATA IS TO BE FACTORED
COL.8 *LOGA**PUNCH 1 IF TRANSFORMED DATA IS TO BE FACTORED
C (BOTH RAW AND TRANSFORMED DATA MAY BE FACTORED DURING SAME JOB)
COL.10*KMIN*MINIMUM VALUE TO POWER MATRIX IN PROMAX ROTATION.BLANK IF NO OBLIQUE
C ROTATION IS DESIRED
COL.12*KMAX*MAXIMUM VALUE TO POWER MATRIX IN PROMAX ROTATION
COL.14*MATR*OPTIONAL INPUT OF A MATRIX.PUNCH 1 IF CORRELATION MATRIX,2 IF
C PRINCIPAL FACTOR MATRIX,3 IF VARIMAX MATRIX,OTHERWISE BLANK
COL.16*IVSCR**PUNCH 1 IF FACTOR SCORES ARE TO BE COMPUTED FROM VARIMAX MATRIX
COL.18*IPSCR**PUNCH 1 IF FACTOR SCORES ARE TO BE COMPUTED FROM PROMAX MATRIX
COL.20*IPTI*FACTOR SCORES.PUNCH 1 FOR CARD OUTPUT,2 FOR TAPE, BLANK PRINT ONLY
COL.22*IOP**PUNCH 1 FOR CARD OUTPUT OF ALL MATRICES
COL.23-26*EV*OPTION ALLOWS NF TO EQUAL THE NUMBER OF FACTORS WITH EIGENVALUES
C EQUAL TO OR GREATER THAN THE VALUE PUNCHED

```

```

C CARD 2**TITLE CARD**PUNCH TITLE OF JOB
C
C CARDS 3,4**VARIABLE NAMES*PUNCH NAMES OF VARIABLES IN FIELDS OF FOUR COLUMNS
C
C CARDS 5,6**INPUT FORMAT**PUNCH INPUT FORMAT OF SAMPLE DATA OR MATRIX
C
C FOLLOW CARD 6 WITH THE DATA CARDS FOR JOB AS SPECIFIED BY INPUT FORMAT
C WHEN MULTIPLE JOBS ARE RUN K BLANK CARDS ARE PLACED BETWEEN LAST DATA CARD
C AND CONTROL CARD FOR NEXT JOB,WHERE K IS THE NUMBER OF CARDS SPECIFIED BY THE
C INPUT FORMAT.FOR LAST OR ONLY JOB K+1 BLANK CARDS FOLLOW DATA CARDS.IF INPUT
C IS A MATRIX K = 0
C
C

```

```

COMMON R(36,36),S(36,36),X(36),Y(36),SD(36),XY(36),NAME(40),NV,NF,
1 NOBS,IOUT,IPUN,LAR,LOGA,IPT
DIMENSION H(36,26),E(27,27),W(36),IFMT(40),JHEAD(20)
EQUIVALENCE (R(937),H(1)),(S(577),E(1)),(S(1),W(1)),(S(1),IFMT(1))
IN = 60
IOUT = 61
IPUN = 62
NTAPE1 = 1
NTAPE2 = 2
40 READ(IN,41) NV,NF,LAR,LOGA,KMIN,KMAX,MATR,IVSCR,IPSCR,IPT,IOP,EV
41 FORMAT(11I2,F4.2)
IF(NV) 1001,1001,42
42 READ(IN,43) (JHEAD(I),I=1,20),(NAME(I),I=1,40),(IFMT(I),I=1,40)
43 FORMAT(20A4)
KHOLD = KMIN

```

```

C MATRIX INPUT OPTION
C
C

```

```

IF(MATR=1)48,44,45
44 NX = NV
GO TO 46
45 NX = NF
46 DO 47 I = 1,NV
47 READ(IN,IFMT) (R(I,J),J=1,NX)
ITRN = 4H NO

```

ITRN = 4H NO

```

NOBS = 0
LAR = 1
LOGA = 0
IF(MATR-2) 101,200,234

```

C SET CORRELATION MATRIX TO ZERO AND READ SAMPLE DATA
C

```

48 NOBS = 0
NOBST = 0
4 DO 5 I = 1,NV
XY(I) = 0.0
5 R(I,J) = 0.0
REWIND NTAPE1
6 IF(LOGA-2) 7,10,10
7 READ(IN,IFMT) ID,(Y(I),I=1,NV)
IF(ID) 20,20,8
8 NOBS = NOBS + 1
WRITE(NTAPE1) ID,(Y(I),I=1,NV)
IF(LAR-LOGA)12,14,14
10 IF(NOBST-NOBS) 11,20,20
11 READ(NTAPE1) ID,(Y(I),I=1,NV)
NOBST = NOBST + 1

```

C DATA TRANSFORMATION
C

```

12 DO 13 I = 1,NV
13 Y(I) = ALOG10(Y(I))
14 DO 15 I = 1,NV
XY(I) = XY(I) + Y(I)
DO 15 J = I,NV
15 R(I,J) = R(I,J)+Y(I)*Y(J)
GO TO 6

```

C FORM CORRELATION MATRIX, COMPUTE MEANS AND STANDARD DEVIATIONS
C

```

20 PNOBS = NOBS
DO 22 I = 1,NV
DO 21 J = I,NV

```

```
21 R(I,J) = R(I,J)-XY(I)*XY(J)/PNOBS
   XY(I) = XY(I)/PNOBS
22 SD(I) = SQRT(R(I,I)/(PNOBS-1.0))
   DO 23 I = 1,NV
   DO 23 J = I,NV
   R(I,J) = R(I,J)/(PNOBS-1.0)
   R(I,J) = R(I,J)/(SD(I)*SD(J))
23 R(J,I) = R(I,J)
   IF(LAR-LOGA) 25,24,24
24 ITRN = 4H NO
   GO TO 26
25 ITRN = 4H LOG
26 IF(IOP) 39,39,36
36 DO 37 I = 1,NV
37 WRITE(IPUN,38) (R(I,J),J=1,NV)
38 FORMAT(10F8.4)
   WRITE(IPUN,38) (XY(J),J=1,NV)
   WRITE(IPUN,38) (SD(J),J=1,NV)
39 JX = 1
   JP = 12
27 IF(JP-NV) 29,28,28
28 JP = NV
29 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
30 FORMAT(1H1,2CX,20A4,10X,A4,2X,14HTRANSFORMATION)
   WRITE(IOUT,34) NOBS,(NAME(I),I=JX,JP)
   DO 32 I = 1,NV
32 WRITE(IOUT,35) NAME(I),XY(I),SD(I),(R(I,J),J=JX,JP)
   IF(JP-NV) 33,101,101
33 JX = JX + 12
   JP = JP + 12
   GO TO 27
34 FORMAT(1H0,9X,16HMEAN      STD DEV,30X,21HCORRELATION MATRIX OF,15,
   1X,7HSAMPLES//25X,12(5X,A4))
35 FORMAT(1H ,1X,A4,2E10.3,12F9.3)
C
C COMMENCE EXTRACTION OF EIGENVALUES AND EIGENVECTORS
C
101 DO 103 I = 1,NV
   DO 102 J = 1,NV
```

5 3 5 3

DO 102 J = 1,NV

```
102 S(I,J) = 0.0
103 S(I,I) = 1.0
    IF(NV-1) 142,142,104
C
C FIND LARGEST OFF-DIAGONAL ELEMENT IN EACH ROW
C
104 NM = NV-1
    DO 106 I = 1,NM
      X(I) = 0.
      NP = I + 1
      DO 106 J = NP,NV
        IF(X(I)-ABSF(R(I,J))) 105,105,106
105 X(I) = ABSF(R(I,J))
      Y(I) = J
106 CONTINUE
C
C TEST FOR END OF PROBLEM
C
RAP = 7.450580596E-09
TST = 1.0E38
107 DO 110 I = 1,NM
  IF(I-1) 109,109,108
108 IF(XMAX-X(I)) 109,110,110
109 XMAX = X(I)
  KP = I
  JP = Y(I)
110 CONTINUE
  IF (XMAX)142,142,111
111 IF(TST) 113,113,112
112 IF(XMAX-TST)113,113,116
113 RM = ABSF(R(I,I))
  DO 115 I = 2,NV
    IF(RM-ABSF(R(I,I))) 115,115,114
114 RM = ABSF(R(I,I))
115 CONTINUE
  TST = RM*RAP
  IF(TST-XMAX)116,142,142
C
C COMPUTE TANGENT,SINE AND COSINE OF R(I,I),R(J,J)
```

```
C
116 RA = R(KP,KP)
    RB = R(JP,JP)
    RC = R(KP,JP)
    TANG=SIGNF(2.,(RA-RB))*RC/(ABSF(RA-RB)+SQRTF((RA-RB)**2+4.*RC**2))
    CSN = 1./SQRTF(1.+TANG**2)
    SNE = TANG*CSN
    R(KP,KP) = CSN**2*(RA+TANG*(2.*RC+TANG*RB))
    R(JP,JP) = CSN**2*(RB-TANG*(2.*RC-TANG*RA))
    R(KP,JP) = 0.0.

C
C PSEUDO RANK THE EIGENVALUES
C
    IF(R(KP,KP)-R(JP,JP)) 117,118,118
117 HT = R(KP,KP)
    R(KP,KP) = R(JP,JP)
    R(JP,JP) = HT
    HT = SIGNF(1.,-SNE)*CSN
    CSN = ABSF(SNE)
    SNE = HT
118 CONTINUE

C
C INSPECT Y(I) BETWEEN I+1 AND NV-1 TO DETERMINE WHETHER A NEW MAXIMUM SHOULD BE
C COMPUTED
C
    DO 126 I = 1,NM
      IF(I-KP) 120,126,119
119 IF(I-JP) 120,126,120
120 IF (Y(I)-KP) 121,122,121
121 IF (Y(I)-JP) 126,122,126
122 K = Y(I)
      HT = R(I,K)
      R(I,K) = 0.0
      NP = I + 1
      X(I) = 0.0

C
C SEARCH IN DERLETED ROW FOR NEW MAXIMUM
C
    DO 125 J = NP,NV
      IF(X(I)-ABSF(R(I,J))) 124,124,125
```

DO 125 J = NP,NV

```
IF(X(I)-ABSF(R(I,J))) 124,124,125
124 X(I) = ABSF(R(I,J))
Y(I) = J
125 CONTINUE
R(I,K) = HT
126 CONTINUE
X(KP) = 0.0
X(JP) = 0.0
```

C CHANGE THE OTHER ELEMENTS OF R
C
C

```
DO 139 I = 1,NV
IF(I-KP) 127,139,131
127 HT = R(I,KP)
R(I,KP) = CSN*HT + SNE*R(I,JP)
IF(X(I)-ABSF(R(I,KP))) 128,129,129
128 X(I) = ABSF(R(I,KP))
Y(I) = KP
129 R(I,JP) = -SNE*HT + CSN*R(I,JP)
IF(X(I)-ABSF(R(I,JP))) 130,139,139
130 X(I) = ABSF(R(I,JP))
Y(I) = JP
GO TO 139
131 IF(I-JP)132,139,135
132 HT = R(KP,I)
R(KP,I) = CSN*HT + SNE*R(I,JP)
IF(X(KP) - ABSF(R(KP,I))) 133,134,134
133 X(KP) = ABSF(R(KP,I))
Y(KP) = I
134 R(I,JP) = -SNE*HT + CSN*R(I,JP)
IF(X(I)-ABSF(R(I,JP))) 130,139,139
135 HT = R(KP,I)
R(KP,I) = CSN*HT + SNE*R(JP,I)
IF(X(KP)-ABSF(R(KP,I))) 136,137,137
136 X(KP) = ABSF(R(KP,I))
Y(KP) = I
137 R(JP,I) = -SNE*HT + CSN*R(JP,I)
IF(X(JP) - ABSF(R(JP,I))) 138,139,139
138 X(JP) = ABSF(R(JP,I))
```

```
Y(JP) = I
139 CONTINUE
DO 141 I = 1,NV
HT = S(I,KP)
S(I,KP) = CSN*HT + SNE*S(I,JP)
141 S(I,JP) = -SNE*HT + CSN*S(I,JP)
GO TO 107

C
C REMOVE EIGENVALUES FROM MATRIX R AND MULTIPLY EACH EIGENVECTOR BY SQUARE ROOT
C OF THE ASSOCIATED EIGENVALUE
C
142 ENV = NV
DO 143 J = 1,NV
Y(J) = R(J,J)
X(J) = SORTF(APSF(Y(J)))
DO 143 I = 1,NV
143 R(I,J) = S(I,J)*X(J)
DO 144 J = 1,NV
144 W(J) = X(J) = Y(J)*100./ENV
DO 145 J = 2,NV
145 W(J) = W(J)+W(J-1)
DO 165 I = 1,NV
165 WRITE(IPUN,38) (R(I,J),J=1,NV)
JX = 1
JP = 12
146 IF(JP-NV) 148,147,147
147 JP = NV
148 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
WRITE(IOUT,160) (J,J=JX,JP)
WRITE(IOUT,161) (Y(J),J=JX,JP)
WRITE(IOUT,162) (X(J),J=JX,JP)
WRITE(IOUT,163) (W(J),J=JX,JP)
DO 152 I = 1,NV
152 WRITE(IOUT,164) NAME(I), (R(I,J),J=JX,JP)
IF(JP-NV) 153,154,154
153 JX = JX + 12
JP = JP + 12
GO TO 146
C
```


C

```

C OPTION ALLOWING NUMBER OF FACTORS FOR ROTATION TO BE DETERMINED BY A MINIMUM
C EIGENVALUE
C
154 IF(EV-0.001) 200,200,155
155 DO 156 J = 1,NV
    IF(Y(J)-EV) 157,156,156
156 CONTINUE
157 NF = J-1
160 FORMAT(IH0,35X,23HUNROTATED FACTOR MATRIX//13X,6HFACOR,2X,12(7X,I
12))
161 FORMAT(IH0,9X,10HEIGENVALUE,1X,12F9.2)
162 FORMAT(IH0,12X,7HPERCENT,12F9.1)
163 FORMAT(IH0,1X,18HCUMULATIVE PERCENT,12F9.1)
164 FORMAT(15X,A4,3X,12F9.3)
C
C COMMENCE VARIMAX ROTATION
C
200 EPS = 0.00116
    ST = 0.7071067812
C
C COMPUTE COMMUNALITY AND NORMALIZE MATRIX
C
DO 202 N = 1,NV
  X(N) = 0.0
DO 201 M = 1,NF
  X(N) = X(N) + R(N,M)**2
201 Y(N) = SQRT(X(N))
DO 202 M = 1,NF
  R(N,M) = R(N,M)/Y(N)
  L = NF - 1
C
C COMPUTE ANGLE OF ROTATION FOR EACH PAIR OF FACTORS
C
203 NOR = 0
DO 221 M = 1,L
  K = M + 1
DO 221 MK = K,NF
  A=B=C=D=0.0
DO 204 N = 1,NV

```

```
U = R(N,M)**2 - R(N,MK)**2
V = R(N,M)*R(N,MK)*2.0
A = A+U
R = B+V
C = C+U**2 - V**2
204 D = D+U*V*2.0
QN = D-2*A*B/NV
QD = C-(A**2-B**2)/NV
C IS FURTHER ROTATION POSSIBLE FOR A GIVEN PAIR OF FACTORS
C
IF(ABSF(QN)+ABSF(QD)) 219,219,205
205 IF(ABSF(QN)-ABSF(QD)) 206,213,210
206 RA = ABSF(QN/QD)
IF(RA-EPS) 208,207,207
207 S4 = COSF(ATANF(RA))
SN = SINF(ATANF(RA))
GO TO 214
208 IF(QD) 209,219,219
209 SNP = SP = ST
GO TO 220
210 RA = ABSF(QD/QN)
IF(RA-EPS) 212,211,211
211 SN = 1.0/SQRTF(1.0+RA**2)
S4 = SN*RA
GO TO 214
212 S4 = 0.0
SN = 1.0
GO TO 214
213 S4 = SN = ST
214 RA = SQRTF((1.0+S4)*0.5)
TH = SQRTF((1.0+RA)*0.5)
TN = SN/(4.0*TH*RA)
IF(QD) 215,216,216
215 SP = ST*(TH+TN)
SNP = ST*(TH-TN)
GO TO 217
216 SP = TH
SNP = TN
```

```
217 IF(QN) 218,220,220
218 SNP = -SNP
```

216 SP = TH
SNP = TN

```
217 IF(QN) 218,220,220
218 SNP = -SNP
   GO TO 220
219 NOR = NOR + 1
   GO TO 221
220 DO 221 N = 1,NV
   RA = R(N,M)*SP + R(N,MK)*SNP
   R(N,MK) = R(N,MK)*SP - R(N,M)*SNP
   R(N,M) = RA
221 CONTINUE
C
C IS ROTATION COMPLETE FOR ALL PAIRS OF FACTORS
C
C IF(NOR-(NF*L)/2) 203,222,203
C
C DENORMALIZE MATRIX AND COMPUTE THE VARIANCE CONTRIBUTED BY EACH FACTOR
C
222 DO 223 N = 1,NV
   DO 223 M = 1,NF
223 R(N,M) = R(N,M)*Y(N)
   RA = 0.0
   DO 225 M = 1,NF
   Y(M) = 0.0
   DO 224 N = 1,NV
224 Y(M) = Y(M) + R(N,M)**2
225 RA = RA + Y(M)
   ENV = NV
   RA = RA*100./ENV
   IF(IOP) 239,239,237
237 DO 238 I = 1,NV
238 WRITE(IPUN,38) (R(I,J),J=1,NV)
239 JX = 1
   JP = 12
226 IF(JP-NF) 228,227,227
227 JP = NF
228 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
   WRITE(IOUT,231) RA,(J,J=JX,JP)
   WRITE(IOUT,232) (Y(J),J=JX,JP)
   DO 229 I = 1,NV
229 WRITE(IOUT,233) X(I),NAME(I),(R(I,J),J=JX,JP)
```

```
IF(JP-NF) 230,234,234
230 JX = JX + 12
      JP = JP + 12
      GO TO 226
231 FORMAT(1H0,25X,30HVARIMAX MATRIX ACCOUNTING FOR ,F5.1,34H PERCENT
      1OF TOTAL PROBLEM VARIANCE//11X,6HFACTOR,2X,12(I7,2X))
232 FORMAT(1H0,15H SUM OF SQUARES,5X,12(F8.3,1X))
233 FORMAT(1H ,3X,F6.3,4X,A4,2X,12F9.3)
234 IF(IVSCR) 290,299,235
235 WRITE(IOUT,236)
236 FORMAT(1H1,35X,56HFACTOR SCORE ROUTINE CALLED FOR THE ABOVE VARIMA
      1X MATRIX)
      CALL FSCOR
299 IF(KMIN) 351,351,300
C
C COMMENCE PROMAX OBLIQUE ROTATION
C MULTIPLY TRANSPOSE OF VARIMAX MATRIX R BY R AND FIND INVERSE OF PRODUCT E
C
300 DO 301 I = 1,NF
      DO 301 J = 1,NF
        E(I,J) = 0.0
      DO 301 K = 1,NV
        E(I,J) = E(I,J) + R(K,I)*R(K,J)
      CALL XINVR
C
C NORMALIZE VARIMAX MATRIX R BY ROWS AND COLUMNS, THEN POWER ELEMENTS OF THE
C RESULTING MATRIX. H BY KMIN, RETAINING THEIR SIGN
C
DO 304 I = 1,NV
  RA = 0.0
DO 303 J = 1,NF
  RA = RA + R(I,J)**2
  RA = SQRTF(RA)
DO 304 J = 1,NF
  H(I,J) = R(I,J)/RA
DO 306 J = 1,NF
  RB = 0.0
DO 305 I = 1,NV
  RB = RB + H(I,J)**2
```

```
RB = SQRTF(RB)
DO 306 I = 1,NV
```

```

DO 305 I = 1,NV
305 RB = RB + H(I,J)**2

RB = SQRTF(RB)
DO 306 I = 1,NV
306 H(I,J) = H(I,J)/RB
DO 307 I = 1,NV
DO 307 J = 1,NF
RA = H(I,J)
RB = ABSF(RA**KMIN)
307 H(I,J) = SIGNF(RB,RA)

C
C MULTIPLY E BY TRANSPOSE OF R AND THE PRODUCT BY H TO OBTAIN TRANSFORMATION
C MATRIX E
C
DO 315 I = 1,NF
DO 308 J = 1,NV
Y(J) = 0.0
DO 308 K = 1,NF
308 Y(J) = Y(J) + E(I,K)*R(J,K)
DO 315 J = 1,NF
E(I,J) = 0.0
DO 315 K = 1,NV
315 E(I,J) = E(I,J) + Y(K)*H(K,J)

C
C NORMALIZE E BY COLUMNS
C
DO 310 J = 1,NF
RB = 0.0
DO 309 I = 1,NF
309 RB = RB + E(I,J)**2
DO 310 I = 1,NF
310 E(I,J) = E(I,J)/RB

C
C FORM OBLIQUE REFERENCE STRUCTURE MATRIX H BY MULTIPLYING R BY E
C
DO 311 I = 1,NV
DO 311 J = 1,NF
H(I,J) = 0.0
DO 311 K = 1,NF
311 H(I,J) = H(I,J)+R(I,K)*E(K,J)
IF(IOP) 362,362,360

```

```
360 DO 361 I = 1,NV
361 WRITE(IPUN,38) (H(I,J),J=1,NF)
362 JX = 1
    JP = I2
312 IF(JP-NF) 314,313,313
313 JP = NF
314 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
    WRITE(IOUT,333) KMIN,(J,J=JX,JP)
    DO 318 I = 1,NV
318 WRITE(IOUT,164) NAME(I), (H(I,J),J=JX,JP)
    IF(JP-NF) 319,320,320
319 JX = JX + I2
    JP = JP + I2
    GO TO 312
333 FORMAT(1H0,35X,49HPROMAX OBLIQUE REFERENCE STRUCTURE MATRIX KMIN
1=>I3//13X,6HFACTOR,2X,12(7X,I2))
C
C COMPUTE INVERSE OF TRANSFORMATION MATRIX E, THEN NORMALIZE E BY ROWS AND
C MULTIPLY RESULTING MATRIX BY ITS TRANSPOSE TO OBTAIN MATRIX OF CORRELATIONS H
C BETWEEN THE OBLIQUE PRIMARY FACTORS
C
320 CALL XINVR
    DO 322 I = 1,NF
    RA = 0.0
    DO 321 J = 1,NF
321 RA = RA + E(I,J)**2
    RA = SORTF(RA)
    DO 322 J = 1,NF
322 E(I,J) = E(I,J)/RA
    DO 323 I = 1,NF
    DO 323 J = 1,NF
    H(I,J) = 0.0
    DO 323 K = 1,NF
323 H(I,J) = H(I,J) + E(I,K)*E(J,K)
    IF(IOP) 365,365,363
363 DO 364 I = 1,NF
364 WRITE(IPUN,38) (H(I,J),J=1,NF)
365 JX = 1
    JP = I2
```

```
324 IF(JP-NF) 326,325,325
325 JP = NF
```

```

324 IF(JP-NF) 326,325,325
325 JP = NF
326 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
    WRITE(IOUT,332) KMIN,(J,J=JX,JP)
    DO 330 I = 1,NF
330 WRITE(IOUT,336) I,(H(I,J),J=JX,JP)
    IF(JP-NF) 331,334,334
331 JX = JX + 12
    JP = JP + 12
    GO TO 324
332 FORMAT(1H0,35X,50HCORRELATIONS BETWEEN PROMAX PRIMARY FACTORS KMI
    1N=I3//13X,6HFACOR,2X,12(I7,2X))
336 FORMAT(15X,12,5X,12F9.3)

C
C  COMPUTE INVERSE OF E, THEN MULTIPLY R BY E TO FORM PRIMARY PATTERN MATRIX H
C
334 CALL XINVR
    DO 337 I = 1,NV
    DO 337 J = 1,NF
        H(I,J) = 0.0
    DO 337 K = 1,NF
337 H(I,J) = H(I,J) + R(I,K)*E(K,J)
    IF(IOP) 368,368,366
366 DO 367 I = 1,NV
367 WRITE(IPUN,38) (H(I,J),J=1,NF)
368 JX = 1
    JP = 12
338 IF(JP-NF) 340,339,339
339 JP = NF
340 WRITE(IOUT,30) (JHEAD(I),I=1,20),ITRN
    WRITE(IOUT,335) KMIN,(J,J=JX,JP)
    DO 344 I = 1,NV
344 WRITE(IOUT,164) NAME(I),(H(I,J),J=JX,JP)
    IF(JP-NF) 345,346,346
345 JX = JX + 12
    JP = JP + 12
    GO TO 338
335 FORMAT(1H0,35X,46HPROMAX OBLIQUE PRIMARY PATTERN MATRIX . KMIN =,I
    13//13X,6HFACOR,2X,12(7X,12))

```

```
C REPEAT PROMAX ROTATION,IF DESIRED,POWERING VARIMAX MATRIX BY KMIN + 1
C
346 IF(KMAX-KMIN) 349,349,347
347 KMIN = KMIN + 1
   GO TO 300
349 IF(IPSCR) 351,351,350
350 WRITE(IOUT,348) KMIN
348 FORMAT(IH1,35X,62HF)FACTOR SCORE ROUTINE CALLED FOR THE ABOVE PROMAX
   IMATRIX KMIN = I3)
   DO 353 I = 1,NV
   DO 353 J = 1,NF
353 R(I,J) = H(I,J)
   CALL FSCOR
C
C REPEAT PROGRAM,IF DESIRED,USING TRANSFORMED DATA
C
351 IF(LAR-LOGA) 40,352,40
352 LOGA = LOGA + 1
   KMIN = Khold
   GO TO 4
1001 REWIND NTAPE1
   REWIND NTAPE2
   WRITE(IOUT,1002)
1002 FORMAT(IH1,18H)THIS IS END OF RUN
   STOP
   END
SUBROUTINE FSCOR
COMMON R(36,36),S(36,36),X(36),Y(36),SD(36),XY(36),NAME(40),NV,NF,
1NOBS,IOUT,IPUN,LAR,LOGA,IPT
DIMENSION H(36,26),E(27,27),ASCOR(26),YS(36)
EQUIVALENCE (R(937),H(1)),(S(577),E(1)),(X(11),ASCOR(1)),(YS,Y)
KOB5 = 0
NTAPE1 = 1
NTAPE2 = 2
REWIND NTAPE1
REWIND NTAPE2
C
C FORM E BY MULTIPLYING TRANSPOSE OF ROTATED FACTOR MATRIX R BY R. THEN FIND THE
```

```
C INVERSE OF E AND MULTIPLY THIS BY TRANSPOSE OF R TO GIVE FACTOR SCORE MATRIX
C
```


C FORM E BY MULTIPLYING TRANSPOSE OF ROTATED FACTOR MATRIX R BY R. THEN FIND THE

```
C INVERSE OF E AND MULTIPLY THIS BY TRANSPOSE OF R TO GIVE FACTOR SCORE MATRIX
C
C H
DO 2 I = 1,NF
DO 2 J = 1,NF
E(I,J) = 0.0
DO 2 K = 1,NV
2 E(I,J) = E(I,J) + R(K,I)*R(K,J)
CALL XINVR
DO 3 I = 1,NF
DO 3 J = 1,NV
H(J,I) = 0.0
DO 3 K = 1,NF
3 H(J,I) = H(J,I) + E(I,K)*R(J,K)
JX = 1
JP = 12
30 IF(JP-NF) 6,5,5
5 JP = NF
6 WRITE(IOUT,8) (J,J=JX,JP)
DO 7 J = 1,NV
7 WRITE(IOUT,9) NAME(J),(H(J,I),I=JX,JP)
8 FORMAT(1H,35X,19HFACOR SCORE MATRIX//13X,6HFACOR,2X,12(I7,2X))
9 FORMAT(15X,A4,3X,12F9.3)
IF(JP-NF) 31,32,32
31 JX=JX+12
JP= JP+12
GO TO 30
32 WRITE(IOUT,10)(J, J=1,NF)
10 FORMAT(1H,35X,21HLIST OF FACTOR SCORES//3X,6HFACOR,14(I7,2X)/9X,
114(I7,2X))
C COMPUTE FOR EACH SAMPLE FROM EITHER RAW OR TRANSFORMED DATA A VECTOR YS OF
C STANDARD SCORES
C
C 20 IF(KORS=NOBS)21,22,22
21 READ(NTAPE)ID,(YS(J),J=1,NV)
KORS = KORS+1
IF(LAR-LOGA)11,13,13
11 DO 12 J = 1,NV
```

```
12 YS(J) = ALOG10(YS(J))
13 DO 14 J= 1,NV
14 YS(J) = (YS(J)-XY(J))/SD(J)
C
C OBTAIN FACTOR SCORE VECTOR ASCOR BY MULTIPLYING H BY YS
C
DO 15 I = 1,NF
  ASCOR(I) = 0.0
DO 15 J = 1,NV
  ASCOR(I) = ASCOR(I) + H(J,I)*YS(J)
  WRITE(IOUT,16) ID,(ASCOR(I),I = 1,NF)
16 FORMAT(1H0,2X,I7,14F9.3/5X,14F9.3)
  IF(IPT-1) 20,17,19
17 WRITE(IPUN,18) ID,(ASCOR(I),I = 1,NF)
18 FORMAT(17,10F7.3/11F7.3/11F7.3)
  GO TO 20
19 WRITE(NTAPE2) ID,(ASCOR(I),I=1,NF)
  GO TO 20
22 RETURN
  END

SUBROUTINE XINVR
COMMON R(36,36),S(36,36),X(36),Y(36),SD(36),XY(36),NAME(40),NV,NF,
1NOBS,IOUT,IPUN,LAR,LOGA,IPT
DIMENSION E(27,27)
EQUIVALENCE (S(577),E(1))
ICOUNT = 1
NK = NF+1
1 E(1,NK) = 1.0
DO 2 I = 2,NF
2 E(I,NK) = 0.0
  T = E(1,I)
  IF(ABS(T)-0.0009) 3,5,5
3 WRITE(61,4)
4 FORMAT(49H1PROGRAM TERMINATED, PIVOT SMALLER THAN LIMIT SET)
  STOP
5 DO 6 J = 1,NF
  K = J + 1
6 Y(J) = E(1,K)/T
```

```
DO 7 I = 2,NF
TT = E(I,1)
```

6 Y(J) = E(I,K)/T

```
DO 7 I = 2,NF
TT = E(I,I)
DO 7 J = 1,NF
JC = J + 1
7 E(I,J) = E(I,JC)-TT*Y(J)
NZ = NF - 1
DO 8 I = 1,NZ
JC = I + 1
DO 8 J = 1,NK
8 E(I,J) = E(JC,J)
DO 9 J = 1,NK
9 E(NF,J) = Y(J)
ICOUNT = ICOUNT + 1
IF(ICOUNT-NF)1,1,10
10 RETURN
END
FINIS
```

Example of output from GEOFACT (Swan Hills carbonates)

CARBONATE SEDIMENTSSWAN HILLS MAIN REEF**SUBSET OF FIFTY SAMPLES**

FLEMENT DATA AS PARTS PER MILLION

SAMPLE NUMBER	TI	FE	SI	AL	MN	MG	RA	SR
10 7966	229.5	1593.9	7835.2	7093.4	67.1	6500.	24.8	374.5
10 7970	87.4	953.4	2065.0	1776.7	50.1	2800.	14.1	193.7
10 7998	23.1	783.3	2607.8	1251.8	25.5	5800.	10.9	315.0
10 8010	2.7	495.6	944.0	592.2	21.8	2900.	8.0	197.9
10 8116	83.0	1358.7	4259.8	2537.2	36.7	13500.	16.3	273.2
10 8140	22.8	537.6	2277.4	982.6	26.0	3900.	6.8	217.1
10 8160	59.2	1092.0	3528.2	2153.6	28.8	10300.	13.7	276.8
10 8172	188.7	1253.7	7198.0	4118.8	31.0	11200.	16.6	307.5
10 8184	45.6	606.9	3138.8	1951.7	21.9	5000.	9.4	319.8
10 8252	154.7	1226.4	6926.6	3863.0	24.6	6700.	14.9	288.1
11 7913	6.1	684.6	944.0	605.7	24.5	5400.	8.8	292.2

11	7918	76.2	1079.4	3209.6	2301.7	23.0	6500.	48.5	175.8
11	7927	3.4	432.6	767.0	444.2	27.2	2800.	4.5	179.9
11	7981	5.0	656.2	919.0	505.9	27.8	8100.	2.8	269.2
11	8001	32.3	1509.0	1814.4	1537.4	37.3	9700.	4.5	291.6
11	8004	102.7	1076.4	3364.2	2595.1	22.6	7300.	10.9	288.7
11	8019	20.7	1093.0	1323.0	683.3	36.1	12000.	4.2	259.7
11	8062	8.6	1755.4	1310.4	558.5	39.4	24700.	4.0	261.4
11	8073	16.6	780.4	1600.2	880.4	27.2	10400.	2.6	244.4
11	8108	55.4	950.1	5556.4	2240.4	25.8	11000.	5.6	279.9
11	8112	200.7	2250.1	7471.8	5111.5	51.4	24300.	15.8	263.1
11	8116	151.6	1169.6	5846.4	4244.2	26.7	8800.	10.6	330.5
11	8154	51.5	1231.6	3540.6	1918.4	16.6	7600.	3.5	301.1
11	8159	228.5	1749.1	7156.8	5019.5	30.9	6700.	13.9	328.8
11	8203	78.7	809.4	3339.0	1984.1	29.8	10600.	6.3	309.7
11	8232	23.4	1496.6	4170.6	998.6	57.4	11000.	4.4	236.0
12	8070	37.6	1486.3	3578.4	1550.5	34.9	12700.	4.9	271.6
12	8161	17.2	1709.8	2192.4	801.5	45.4	18500.	4.6	215.4
12	8283	124.8	2017.2	6085.3	2957.1	55.1	2500.	13.1	1346.1
13	7893	29.2	2463.0	1494.9	899.0	57.4	20900.	3.4	179.7
13	7907	60.0	4186.2	1419.4	1263.1	99.4	33600.	2.4	139.1
13	7912	19.2	2318.4	1238.2	609.3	61.1	25200.	6.4	188.5
13	7920	75.2	1397.8	2808.6	1738.6	98.4	16500.	7.6	203.2
13	7969	114.8	1352.0	6568.5	2555.9	35.1	18000.	9.4	256.3
13	7959	26.0	1571.3	1963.0	884.2	38.7	20500.	3.7	231.7
13	7964	49.6	1426.7	3759.9	1538.0	33.6	18500.	10.1	277.1
13	8070	82.0	2171.4	3790.1	2771.4	90.9	6300.	6.6	327.4
14	8040	81.6	686.8	2914.3	2496.5	22.7	5300.	6.9	278.8
14	8025	40.8	1048.3	2340.5	1337.4	36.3	6300.	7.1	299.8
14	8282	21.3	796.5	1010.1	722.6	38.4	8200.	3.1	223.5
14	8292	16.8	440.3	1343.1	260.4	62.6	4200.	5.1	199.1
15	7873	16.4	1824.4	1072.1	787.6	35.5	22600.	4.9	318.8
15	7893	36.3	762.5	1332.0	989.5	24.5	10200.	3.8	233.7
15	7898	2.4	1351.5	1032.3	208.3	30.7	16200.	6.8	164.5
15	7908	51.7	1543.0	1898.1	1119.7	35.6	15200.	6.3	207.3
15	8120	57.6	869.9	6340.3	1940.0	55.4	7200.	6.0	282.3
16	8355	27.5	773.5	2419.8	1401.7	23.7	7100.	11.4	243.8
16	8440	15.1	918.8	2583.3	1048.0	36.1	8400.	4.6	255.4
16	8450	4.8	280.0	872.0	478.1	21.9	5300.	3.4	291.6
18	7930	29.6	1076.5	1839.2	1903.7	29.5	13800.	12.1	242.5

ID 4470 4.0 CUMUL 470.1 3.4 7500. 3.4 242.5
 LR 7930 29.6 1076.5 1839.2 1903.7 29.5 1380.0 12.1 242.5

LOG TRANSFORMATION

CARBONATE SEDIMENTSSWAN HILLS MAIN REF**SUBSET OF FIFTY SAMPLES**

MEAN	STD DEV	CORRELATION MATRIX OF 50 SAMPLES									
		TI	FE	SI	AL	MN	MG	HA	SR		
11 1.552E 00 4.976E-01		1.000	.448	.857	.919	.219	.075	.548	.369		
FE 3.047E 00 2.219E-01		.448	1.000	.335	.350	.591	.663	.074	.059		
SI 3.394E 00 2.946E-01		.857	.335	1.000	.845	.098	-.003	.591	.469		
AL 3.134E 00 3.330E-01		.919	.350	.845	1.000	.043	-.034	.671	.659		
MN 1.544E 00 1.801E-01		.219	.591	.098	.043	1.000	.325	-.0105	-.073		
MG 3.967E 00 2.732E-01		.075	.663	-.003	-.034	.325	1.000	-.0269	-.331		
HA 8.507E-01 2.724E-01		.548	.074	.591	.631	-.0105	-.0269	1.000	.257		
SR 2.416E 00 1.377E-01		.369	.059	.469	.659	-.073	-.331	.257	1.000		

LOG TRANSFORMATION

CARBONATE SEDIMENTSSWAN HILLS MAIN REF**SUBSET OF FIFTY SAMPLES**

		UNROTATED FACTOR MATRIX									
FACTOR		1	2	3	4	5	6	7	8		
EIGENVALUE		3.66	2.15	.81	.64	.40	.17	.12	.06		
PERCENT		45.8	26.8	10.1	8.0	5.0	2.2	1.5	.7		
CUMULATIVE PERCENT		45.8	72.6	82.7	90.7	95.6	97.8	99.3	100.0		
TI		.939	.071	-.0074	-.017	.233	-.0090	-.0152	-.0144		
FE		.481	.741	.078	.095	-.0234	-.0223	.150	-.0225		
SI		.930	-.0076	-.040	.060	-.102	.223	.215	-.0026		
AL		.952	-.0106	-.0100	.064	-.146	-.0112	-.0050	.142		
MN		.195	.710	.390	-.0336	.053	.105	-.0051	.035		
MG		.025	.853	-.0263	.381	-.0085	.184	-.0122	.020		
HA		.640	-.0344	-.0362	-.0308	-.0430	.061	-.0046	-.0009		
SR		.526	-.0380	.059	.301	-.0214	.054	-.0072	-.0005		

LOG TRANSFORMATION

CARBONATE SEDIMENTSSWAN HILLS MAIN REF**SUBSET OF FIFTY SAMPLES**

		VARIAM MATRIX ACCOUNTING FOR 99.3 PERCENT OF TOTAL PROBLEM VARIANCE						
FACTOR		1	2	3	4	5	6	7
SUM OF SQUARES		2.858	1.381	1.035	1.191	.943	.345	.150
.979	TI	.940	.088	.119	-.0154	-.0170	-.0068	-.0124
.959	FE	.284	.608	.074	-.0431	-.0045	-.0597	.002
.959	SI	.874	.047	.224	-.0021	-.0014	.363	.363
.967	AL	.924	.010	.194	.017	-.0243	-.0116	-.0046
.999	MN	.067	.175	-.0035	-.0097	.054	-.0086	.003
1.000	MG	.010	.964	-.0181	-.0141	.123	-.0054	.003
1.000	HA	.434	-.0131	-.063	.064	-.0085	-.0019	.021
1.000	SR	.283	-.0159	.943	.031	-.0057	-.0027	.018

FACTOR SCORE ROUTINE CALLED FOR THE ABOVE VARIMAX MATRIX

FACTOR SCORE MATRIX

FACTOR	1	2	3	4	5	6	7
II	.579	.003	-0.133	-0.070	.258	.393	-1.332
FE	-0.132	-0.087	-0.041	.130	.007	-1.908	.271
SI	.289	-0.043	-0.126	.010	.196	.082	2.235
AL	.477	-0.104	-0.126	.157	.208	-0.159	-0.687
MN	-0.061	-0.112	.050	-1.090	-0.110	.633	-0.021
MG	-0.056	1.182	.238	.100	-0.211	1.137	-0.211
BA	-0.305	.184	.054	-0.088	-1.318	.220	-0.141
SR	-0.222	.249	1.180	-0.051	-0.078	.341	-0.275

LIST OF FACTOR SCORES

FACTOR	1	2	3	4	5	6	7
1007966	1.433	-0.540	.675	-1.646	-1.571	.266	-0.179
1007970	.501	-2.369	-1.552	-1.200	-0.847	-0.894	-1.487
1007998	-0.419	-0.448	.612	.613	-0.821	.201	.467
1008010	-1.725	-1.925	-0.827	.880	-0.769	-0.894	.755
1008116	.510	.829	.081	-0.030	-1.382	.566	-0.014
1008140	-0.014	-1.524	-0.864	.478	.278	.383	.613
1008160	.349	.416	.102	.515	-1.043	.372	-0.080
1008172	1.421	.539	.166	.398	-0.875	.650	.248
1008184	.443	-0.744	.415	.995	-0.099	.782	-0.169
1008252	1.448	-0.433	-0.266	.943	-0.472	-0.716	.663

1107913 -1.773 -0.511 .756 .633 -1.111 -0.072 -0.615

1008252	1.448	-0.433	-0.266	.943	-0.472	-0.716	.663
1107913	-1.773	-0.511	.756	.633	-1.111	-0.072	-0.615
1107918	.256	-0.373	-1.701	.923	-3.327	-0.743	-0.476
1107927	-1.523	-2.219	-1.148	.278	.330	-0.302	.200
1107981	-1.475	-0.144	.603	.501	1.012	.475	-0.527
1108001	-0.061	-0.022	.432	.014	.819	-1.103	-0.881
1108004	.875	-0.218	-0.006	1.123	-0.232	-0.457	-0.909
1108019	-0.742	.459	.340	-0.063	.515	.265	-0.803
1108062	-1.495	1.727	.743	-0.040	.101	-0.338	.378
1108073	-0.198	.089	-0.040	.718	1.731	.494	-0.102
1108108	1.007	.305	.005	.922	.967	.565	1.560
1108112	1.442	1.588	-0.161	-0.587	-1.036	.433	.276
1108116	1.470	.071	.351	.803	.068	.060	-0.120
1108154	.891	-0.346	.140	2.109	1.992	-1.879	.625
1108159	1.638	-0.508	.125	.452	-0.246	-1.493	.160
1108203	.787	.387	.482	.410	.565	1.551	-0.612
1108232	-0.052	-0.069	-0.303	-1.121	.813	-0.342	2.742
1208070	.308	.430	.124	.230	.818	-0.574	1.120
1208161	-0.622	.960	-0.291	-0.414	.317	-0.479	1.156
1208283	-0.059	-1.372	5.238	-1.482	-0.618	-1.549	-0.028
1307893	-0.311	.842	-0.955	-0.862	.939	-1.452	-0.459

1307907	.331	1.148	-1.863	-1.994	1.674	-1.956	-1.378
1307912	-1.197	1.474	-0.491	-1.165	-0.737	-0.606	-0.525
1307920	.456	.554	-0.742	-2.508	-0.279	1.737	-0.608
1307959	-0.308	1.198	-0.056	.012	.852	-0.111	.138
1307964	-0.081	1.381	.362	.256	-0.749	.578	.642
1307969	1.159	1.143	-0.220	.207	-0.094	.937	1.112
1308070	.636	-1.058	.470	-2.303	.481	-1.336	.040
1408025	-0.027	-0.604	.385	-0.184	.090	-0.234	-0.363
1408040	1.064	-0.891	-0.200	1.020	.734	.364	-1.078
1408282	-0.449	-0.416	-0.318	-0.294	1.241	.551	-1.620
1408292	-1.105	-1.528	-0.709	-2.078	.062	2.517	.382
1507873	-1.265	1.765	1.355	.197	-0.136	-0.391	-1.504
1507893	.062	.159	-0.268	.911	1.133	.693	-1.768
1507898	-2.649	.986	-0.749	.356	-1.319	-1.078	2.257
1507908	.035	.703	-0.658	.128	.092	-0.461	-0.912
1508120	.903	-0.646	-0.023	-1.218	.767	1.388	2.122
1608355	-0.138	-0.245	-0.294	.881	-0.852	.269	.057
1608440	-0.306	-0.269	.007	-0.057	.683	.167	1.407
1608450	-1.363	-0.608	.798	.796	.772	2.684	-0.632
1807930	-0.229	.877	-0.065	.541	-1.232	.514	-1.274

CARBONATE SEDIMENTSSWAN HILLS MAIN REEF**SUBSET OF FIFTY SAMPLES**
 LOG TRANSFORMATION

PHOMAX OBLIQUE REFERENCE STRUCTURE MATRIX KMIN= 2

FACTOR	1	2	3	4	5	6	7
TI	.785	.027	.008	-0.085	-0.023	.014	-0.062
FE	.071	.324	.004	-0.185	-0.001	-0.520	-0.002
SI	.655	.006	.045	-0.013	-0.046	.004	.411
AL	.735	-0.026	.041	.077	-0.066	-0.052	.009
MN	.000	.022	.000	-0.080	.002	-0.017	.003
MG	-0.002	.804	-0.029	-0.010	.014	-0.004	.002
BA	.164	-0.013	.003	-0.000	-0.754	.001	.001
SK	.069	-0.021	.807	-0.001	-0.003	-0.000	.001

CARBONATE SEDIMENTSSWAN HILLS MAIN REEF**SUBSET OF FIFTY SAMPLES**
 LOG TRANSFORMATION

CORRELATIONS BETWEEN PHOMAX PRIMARY FACTORS KMIN= 2

FACTOR	1	2	3	4	5	6	7
1	1.000	.050	.327	-0.127	-0.405	-0.404	.108
2	.050	1.000	-0.290	-0.282	.248	-0.318	-0.015
3	.327	-0.290	1.000	.079	-0.148	-0.213	.199
4	-0.127	-0.282	.079	1.000	-0.138	.360	.034
5	-0.405	.248	-0.148	-0.138	1.000	.143	-0.159
6	-0.404	-0.318	-0.213	.360	.143	1.000	-0.003
7	.108	-0.015	.199	.034	-0.159	-0.003	1.000

CARBONATE SEDIMENTSSWAN HILLS MAIN REEF**SUBSET OF FIFTY SAMPLES**
 LOG TRANSFORMATION

PHOMAX OBLIQUE PRIMARY PATTERN MATRIX KMIN = 2

FACTOR	1	2	3	4	5	6	7
TI	.971	.033	.010	-0.095	-0.028	.022	-0.064
FE	.088	.395	.005	-0.207	-0.001	-0.654	-0.002
SI	.810	.007	.053	-0.015	-0.055	.005	.426
AL	.909	-0.031	.049	.086	-0.078	-0.064	.009
MN	.000	.026	.000	-0.483	.002	-0.021	.003
MG	-0.002	.979	-0.035	-0.012	.016	-0.010	.002
BA	.203	-0.016	.004	-0.000	-0.896	.001	.001
SK	.085	-0.026	.961	-0.002	-0.003	-0.000	.001

Program listing for GEOSCORE

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PROGRAM GEOSCORE
PROGRAMMED BY E M CAMERON GEOLOGICAL SURVEY OF CANADA    FEB 1967
PROGRAMMED FOR CDC 3200 WITH 16K MEMORY.MAXIMUM OF TWO TAPE DRIVES REQUIRED
PROGRAMMING LANGUAGE *FORTRAN II*
PROGRAM COMPUTES FACTOR SCORES FROM AN ORTHOGONAL OR OBLIQUE FACTOR MATRIX
DERIVED BY ROTATION FROM A PRINCIPAL COMPONENTS MATRIX THAT WAS OBTAINED BY
FACTORING A CORRELATION MATRIX WITH UNITIES IN THE DIAGONAL.
THE RAW SAMPLE DATA MAY BE LOGARITHMICALLY TRANSFORMED PRIOR TO THEIR
CONVERSION TO STANDARD SCORES.
PROGRAM HANDLES JOBS WITH UP TO 36 VARIABLES AND 26 FACTORS
*****CARD INPUT AS FOLLOWS*****
CARD 1**CONTROL CARD**
COL.1-5*NOBS*NUMBER OF SAMPLES
COL.6-7*NV*NUMBER OF VARIABLES
COL.8-9*NF*NUMBER OF FACTORS
COL.11*LOGA*PUNCH 1 IF STANDARD SCORES ARE TO BE COMPUTED FROM LOGARITHMICALLY
TRANSFORMED DATA.
COL.13*KOBL*BLANK IF FACTOR SCORES ARE TO BE COMPUTED FROM AN ORTHOGONAL FACTOR
MATRIX.PUNCH 1 IF THEY ARE TO BE COMPUTED FROM AN OBLIQUE MATRIX
COL.15*INTAPE*BLANK IF INPUT MEDIUM FOR SAMPLE DATA IS CARDS.PUNCH 1 FOR TAPE
COL.17*IPT*PUNCH 1 FOR CARD OUTPUT OF FACTOR SCORES,2 FOR TAPE,BLANK PRINT ONLY
CARD 2**TITLE CARD**PUNCH TITLE OF JOB
CARDS 3,4**VARIABLE NAMES**PUNCH NAMES OF VARIABLES IN FIELDS OF FOUR COLUMNS
CARDS 5,6**INPUT FORMAT**PUNCH INPUT FORMAT OF SAMPLE DATA
C FOLLOW CARD 6 WITH EITHER THE ORTHOGONAL FACTOR MATRIX(NO.OF CARDS=NV) OR
C WITH THE MATRIX OF CORRELATIONS BETWEEN OBLIQUE PRIMARY FACTORS(NO.OF CARDS

```

```

=NF) AND THE OBLIQUE PRIMARY PATTERN MATRIX(NO.OF CARDS=NV).THESE ARE FOLLOWE
D BY THE CORRELATION MATRIX(NO.OF CARDS=NV).THE MEANS THEN THE STANDA

```

C WITH THE MATRIX OF CORRELATIONS BETWEEN OBLIQUE PRIMARY FACTORS(NO.OF CARDS

C =NF) AND THE OBLIQUE PRIMARY PATTERN MATRIX(NO.OF CARDS=NV). THESE ARE FOLLOWE
 C D BY THE CORRELATION MATRIX(NO.OF CARDS=NV),THE MEANS,THEN THE STANDARD
 C DEVIATIONS OF THE RAW OR TRANSFORMED DATA.ALL THESE CARDS HAVE A FORMAT OF
 C 10F8.4.THE SAMPLE DATA FOLLOW IF THE INPUT MEDIUM IS CARDS. CONTROL CARD FOR
 C THE SUCCEEDING JOB IS NEXT,OR A BLANK CARD WHERE PREVIOUS JOB IS LAST.
 C
 C

```

COMMON E(37,37),Y(37),NV
DIMENSION D(36,26),H(36,26),XY(36),SD(36),YS(36),ASCOR(26),JHEAD(2
10),NAME(40),IFMT(40)
EQUIVALENCE(D(1),XY(1)),(D(37),SD(1)),(D(73),YS(1)),(D(109),ASCOR(
11))

```

```

NTAPE1 = 1
NTAPE2 = 2
REWIND NTAPE1
REWIND NTAPE2
IN = 60
IOUT = 61
IPUN = 62

```

```

105 READ(IN,106) NOBS,NV,NF,LOGA,KOBL,INTAPE,IPT
106 FORMAT(15,6I2)

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```

IF(NOBS)100,100,101

```

```

101 READ(IN,107)(JHEAD(I),I=1,20),(NAME(I),I=1,40),(IFMT(I),I=1,40)

```

```

107 FORMAT(20A4)

```

```

108 FORMAT(10F8.4)

```

```

IF(LOGA) 102,102,103

```

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102 ITRN = 4H NO

```

```

GO TO 104

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```

103 ITRN =4H LOG

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```

104 IF(KOBL) 114,114,110

```

C FOR OBLIQUE FACTORS MULTIPLY MATRIX H OF CORRELATIONS BETWEEN THE PRIMARY
 C FACTORS BY THE PRIMARY PATTERN MATRIX E TO OBTAIN MATRIX D
 C

```

110 DO 111 M = 1,NF

```

```

111 READ(IN,108)(H(M,N),N=1,NF)

```

```

DO 112 M = 1,NV

```

```

112 READ(IN,108)(E(M,N),N=1,NF)

```

```

DO 113 I = 1,NF

```

```

DO 113 J = 1,NV

```

```
D(J,I) = 0.0
DO 113 K = 1,NF
113 D(J,I) = D(J,I) + H(I,K)*E(J,K)
GO TO 116
C
C FOR ORTHOGONAL FACTORS READ IN MATRIX D
C
114 DO 115 M = 1,NV
115 READ(IN,108) (D(M,N),N=1,NF)
C
C READ CORRELATION MATRIX E AND COMPUTE ITS INVERSE E
C
116 DO 117 M = 1,NV
117 READ(IN,108) (E(M,N),N=1,NV)
CALL XINVR
C
C MULTIPLY MATRIX D BY INVERSE OF CORRELATION MATRIX E TO PRODUCE FACTOR SCORE
C MATRIX H
C
DO 118 I = 1,NF
DO 118 J = 1,NV
H(J,I) = 0.0
DO 118 K = 1,NV
118 H(J,I) = H(J,I) + D(K,I)*E(K,J)
WRITE(IOUT,119)(JHEAD(I),I=1,20),ITRN
119 FORMAT(1H1,20X,20A4,10X,A4,2X,14HTRANSFORMATION)
JX = 1
JP = 12
30 IF(JP-NF) 6,5,5
5 JP = NF
6 WRITE(IOUT,8) (J,J=JX,JP)
DO 7 J = 1,NV
7 WRITE(IOUT,9) NAME(J),(H(J,I),I=JX,JP)
8 FORMAT(1H0,35X,19HFACTOR SCORE MATRIX//13X,6HFACTOR,2X,12(I7,2X))
9 FORMAT(15X,A4,3X,12F9.3)
IF(JP-NF) 31,32,32
31 JX=JX+12
JP= JP+12
GO TO 30
```

32 WRITE(IOUT,119)(JHEAD(I),I=1,20),ITRN
WRITE(IOUT,119) (JHEAD(I),I=1,20),ITRN

JP= JP+12
GO TO 30

```

32 WRITE(IOUT,119)(JHEAD(I),I=1,20),ITRN
   WRITE(IOUT,10)(J, J=1,NF)
10 FORMAT(1H0,35X,21HLIST OF FACTOR SCORES//3X,6HFACTOR,14(I7,2X)/9X,
   114(I7,2X))

C
C COMPUTE FOR EACH SAMPLE FROM EITHER RAW OR TRANSFORMED DATA A VECTOR YS OF
C STANDARD SCORES
C
   READ(IN,108)(XY(J),J=1,NV)
   READ(IN,108)(SD(J),J=1,NV)
   DO 20 N= 1, NORBS
   IF(NTAPE) 23,23,24
23 READ(IN,IFMT)ID,(YS(J),J=1,NV)
   GO TO 25
24 READ(NTAPE1) ID,(YS(J),J=1,NV)
25 IF(LOGA) 13,13,11
11 DO 12 J = 1,NV
12 YS(J) = ALOG10(YS(J))
13 DO 14 J= 1,NV
14 YS(J) = (YS(J)-XY(J))/SD(J)

C
C OBTAIN FACTOR SCORE VECTOR ASCOR BY MULTIPLYING H BY YS
C
   DO 15 I = 1,NF
   ASCOR(I) = 0.0
   DO 15 J = 1,NV
15 ASCOR(I) = ASCOR(I) + H(J,I)*YS(J)
   WRITE(IOUT,16) ID,(ASCOR(I),I = 1,NF)
16 FORMAT(1H0,2X,I7,14F9.3/5X,14F9.3)
   IF(IPT-1) 20,17,19
17 WRITE(IPUN,18) ID,(ASCOR(I),I = 1,NF)
18 FCRMAT(I7,10F7.3/11F7.3/11F7.3)
   GO TO 20
19 WRITE(NTAPE2) ID,(ASCOR(I),I=1,NF)
20 CONTINUE
   GO TO 105
100 REWIND NTAPE1
   REWIND NTAPE2
   WRITE(IOUT,1002)

```

```
1002 FORMAT(1H1,18HTHIS IS END OF RUN)
STOP
END
SUBROUTINE XINVR
COMMON E(37,37),Y(37),NV,
ICOUNT = 1
NK = NV+1
1 E(1,NK) = 1.0
DO 2 I = 2,NV
2 E(I,NK) = 0.0
T = E(1,1)
IF(ABS(T)-0.0009) 3,5,5
3 WRITE(61,4)
4 FORMAT(49H1PROGRAM TERMINATED, PIVOT SMALLER THAN LIMIT SET)
STOP
5 DO 6 J = 1,NV
K = J + 1
6 Y(J) = E(1,K)/T
DO 7 I = 2,NV
TT = E(I,1)
DO 7 J = 1,NV
JC = J + 1
7 E(I,J) = E(I,JC)-TT*Y(J)
NZ = NV - 1
DO 8 I = 1,NZ
JC = I + 1
DO 8 J = 1,NK
8 E(I,J) = E(JC,J)
DO 9 J = 1,NK
9 E(NV,J) = Y(J)
ICOUNT = ICOUNT + 1
IF(ICOUNT-NV)1,1,10
10 RETURN
END
FINIS
```

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